Unit 12 - Week 10: Ordinary Differential Equations – Initial Value Problems (ODE-IVP)

Assignment 10

The due date for submitting this assignment has passed.
As per our records you have not submitted this assignment.

Due on 2019-10-09, 23:59 IST.

Overall Problem Statement

In this assignment, we will solve the ODE-IVP problem for cooling of a hot ball exposed to colder atmosphere. Newton's Law of Cooling gives:

$$\frac{dT}{dt} = -\frac{ha}{mc_p}(T - T_a)$$

where ha is the net heat transfer coefficient, m is the mass of the ball and c_p is the specific heat. Let us say the ambient is at 30 deg C. For the conditions of interest, when we substitute the typical parameter values, we get

$$\frac{dT}{dt} = -0.5(T - 30), \quad [T]_{t=0} = 80$$

We will solve this ODE-IVP in multiple ways to get T at time $t = 2$

Problem-1: Euler's Method

1) Compute the value of $T(2)$ using a single step of Euler's Explicit method (i.e., using $h = 2$)

No, the answer is incorrect.
Score: 0
Accepted Answers:
(Type: Range) 29.5,30.5

0.2 points

2) Next, we will solve the ODE using two steps of Euler's Explicit method. First, with h = 1, use Euler's explicit method to obtain $T(1)$

No, the answer is incorrect.
Score: 0
Accepted Answers:
(Type: Range) 54.5,55.5

0.2 points

3) Repeat this to obtain $T(2)$

No, the answer is incorrect.
Score: 0
Accepted Answers:
(Type: Range) 42,43

0.2 points

4) Next, we will solve the ODE using four steps of Euler's Explicit method. Thus, with $h = 0.5$, use Euler's explicit method to obtain $T(0.5)$

No, the answer is incorrect.
Score: 0
Accepted Answers:
(Type: Range) 66.5,68.5

0.2 points

5) Repeat this process three more times. Thus, with h = 0.5, compute $T(1), T(1.5)$, and $T(2)$. In the blank below, please report the value of $T(2)$

No, the answer is incorrect.
Score: 0
Accepted Answers:
(Type: Range) 44.8,46.8

0.2 points

Problem 2: RK-2 Midpoint Method

We will now solve the above problem using single application (i.e., with $h = 2$) of RK-2 Midpoint Method, which is given as:

$y_{i+1} = y_i + hk_2$,
where $k_1 = f(y_i, t_i), \quad k_2 = f\left(y_i + \frac{hk_1}{2}, t_i + \frac{h}{2}\right)$

6) Compute the value of k_1

No, the answer is incorrect.
Score: 0
Accepted Answers:
(Type: Range) -25.5,-24.5

0.3 points

7) Compute the value of k_2

No, the answer is incorrect.
Score: 0
Accepted Answers:
(Type: Range) -13,-12

0.3 points

8) Hence compute the value of $T(2)$

No, the answer is incorrect.
Score: 0
Accepted Answers:
(Type: Range) 54.5,55.5

0.4 points

Problem 3: Multiple Applications of Midpoint Method

9) We will now use multiple steps of Midpoint Method to improve the accuracy. In this problem, we will use step-size of $h = 1$. With $h = 0.5$ and $T(0) = 80$, compute the values of k_1 and k_2. Use them to compute the value of $T(0.5)$. Please report the value of $T(0.5)$ in the blank below

No, the answer is incorrect.
Score: 0
Accepted Answers:
(Type: Range) 68,70

0.25 points

10) With the value of $T(0.5)$ so obtained, use it to compute the value of $T(1)$. Please report the value of $T(1)$ in the blank below

No, the answer is incorrect.
Score: 0
Accepted Answers:
(Type: Range) 59.5,61.5

0.25 points

11) Continue the process again and report the value of $T(1.5)$ below

No, the answer is incorrect.
Score: 0
Accepted Answers:
(Type: Range) 52.8,54.8

0.25 points

12) Continue the process again and report the value of $T(2)$ below

No, the answer is incorrect.
Score: 0
Accepted Answers:
(Type: Range) 47.6,49.6

0.25 points

Problem 4: RK-4 Classic Method

We will now solve the above problem using single application (i.e., with $h = 2$) of the Classical RK-4 Method, which is given as:

$y_{i+1} = y_i + \frac{h}{6}\left(k_1 + 2k_2 + 2k_3 + k_4\right)$, where

$k_1 = f(y_i, t_i) \quad k_2 = f\left(y_i + \frac{hk_1}{2}, t_i + \frac{h}{2}\right) \quad k_3 = f\left(y_i + \frac{hk_2}{2}, t_i + \frac{h}{2}\right) \quad k_4 = f(y_i + hk_3, t_i + h)$

13) Compute the value of k_1

No, the answer is incorrect.
Score: 0
Accepted Answers:
(Type: Range) -25.5,-24.5

0.2 points

14) Compute the value of k_2

No, the answer is incorrect.
Score: 0
Accepted Answers:
(Type: Range) -13,-12

0.2 points

15) Compute the value of k_3

No, the answer is incorrect.
Score: 0
Accepted Answers:
(Type: Range) -19.5,-18

0.2 points

16) Compute the value of k_4

No, the answer is incorrect.
Score: 0
Accepted Answers:
(Type: Range) -6.5,-6

0.2 points

17) Hence compute the value of $T(2)$

No, the answer is incorrect.
Score: 0
Accepted Answers:
(Type: Range) 47.75,49.75

0.2 points