

# Unit 12 - Week 10: Ordinary Differential Equations – Initial Value Problems (ODE-IVP)

## Course outline

### How to access the portal?

### Course Pre-requisites and Introduction

### Week 1 - Computation and Error Analysis

### Week 2 - Linear Systems and Equations

### Week 3 - Linear Equations - 2

### Week 4: Nonlinear Equations in Single Variable

### Week 5: Nonlinear equations in Single and Multiple Variables

### Week 6: Regression (Curve Fitting)

### Week 7: Interpolation

### Week 8: Numerical Differentiation

### Week 9: Numerical Integration

### Week 10: Ordinary Differential Equations – Initial Value Problems (ODE-IVP)

- Introduction to ODE-IVP
- Motivation using an Example (Bonus)
- Euler's Methods and Second-Order Methods
- Second-Order Runge-Kutta Methods
- Summary of RK-2
- Higher order RK Methods
- Bonus: ODE-IVP using MS-Excel
- Bonus: RK-2 and RK-4 Methods using MS-Excel
- Summary and Recap

### Quiz : Assignment 10

- Numerical Methods for Engineers : Week 10 Feedback form
- Solutions to Assignment-10

### Week 11: ODE-IVP (Part-2)

### Week 12: ODE - Boundary Value Problems

### Video Download, Live Session and Other Information

### Info about our Final Exam

## Assignment 10

The due date for submitting this assignment has passed. As per our records you have not submitted this assignment.

**Due on 2019-10-09, 23:59 IST.**

### Overall Problem Statement

In this assignment, we will solve the ODE-IVP problem for cooling of a hot ball exposed to colder atmosphere. Newton's Law of Cooling gives:

$$\frac{dT}{dt} = -\frac{ha}{mc_p}(T - T_a)$$

where  $ha$  is the net heat transfer coefficient,  $m$  is the mass of the ball and  $c_p$  is the specific heat. Let us say the ambient is at 30 deg C. For the conditions of interest, when we substitute the typical parameter values, we get

$$\frac{dT}{dt} = -0.5(T - 30), \quad [T]_{t=0} = 80$$

We will solve this ODE-IVP in multiple ways to get  $T$  at time  $t = 2$

### Problem-1: Euler's Method

- 1) Compute the value of  $T(2)$  using a single step of Euler's Explicit method (i.e., using  $h = 2$ )

No, the answer is incorrect. Score: 0

Accepted Answers: (Type: Range) 29.5,30.5

0.2 points

- 2) Next, we will solve the ODE using two steps of Euler's Explicit method. First, with  $h = 1$ , use Euler's explicit method to obtain  $T(1)$

No, the answer is incorrect. Score: 0

Accepted Answers: (Type: Range) 54.5,55.5

0.2 points

- 3) Repeat this to obtain  $T(2)$

No, the answer is incorrect. Score: 0

Accepted Answers: (Type: Range) 42,43

0.2 points

- 4) Next, we will solve the ODE using four steps of Euler's Explicit method. Thus, with  $h = 0.5$ , use Euler's explicit method to obtain  $T(0.5)$

No, the answer is incorrect. Score: 0

Accepted Answers: (Type: Range) 66.5,68.5

0.2 points

- 5) Repeat this process three more times. Thus, with  $h = 0.5$ , compute  $T(1)$ ,  $T(1.5)$ , and  $T(2)$ . In the blank below, please report the value of  $T(2)$

No, the answer is incorrect. Score: 0

Accepted Answers: (Type: Range) 44.8,46.8

0.2 points

### Problem 2: RK-2 Midpoint Method

We will now solve the above problem using single application (i.e., with  $h = 2$ ) of RK-2 Midpoint Method, which is given as:

$$y_{i+1} = y_i + hk_2,$$

$$\text{where } k_1 = f(y_i, t_i), \quad k_2 = f\left(y_i + \frac{hk_1}{2}, t_i + \frac{h}{2}\right)$$

- 6) Compute the value of  $k_1$

No, the answer is incorrect. Score: 0

Accepted Answers: (Type: Range) -25.5,-24.5

0.3 points

- 7) Compute the value of  $k_2$

No, the answer is incorrect. Score: 0

Accepted Answers: (Type: Range) -13,-12

0.3 points

- 8) Hence compute the value of  $T(2)$

No, the answer is incorrect. Score: 0

Accepted Answers: (Type: Range) 54.5,55.5

0.4 points

### Problem 3: Multiple Applications of Midpoint Method

- 9) We will now use multiple steps of Midpoint Method to improve the accuracy. In this problem, we will use step-size of  $h = 1$ . With  $h = 0.5$  and  $T(0) = 80$ , compute the values of  $k_1$  and  $k_2$ . Use them to compute the value of  $T(0.5)$ . Please report the value of  $T(0.5)$  in the blank below

No, the answer is incorrect. Score: 0

Accepted Answers: (Type: Range) 68,70

0.25 points

- 10) With the value of  $T(0.5)$  so obtained, use it to compute the value of  $T(1)$ . Please report the value of  $T(1)$  in the blank below

No, the answer is incorrect. Score: 0

Accepted Answers: (Type: Range) 59.5,61.5

0.25 points

- 11) Continue the process again and report the value of  $T(1.5)$  below

No, the answer is incorrect. Score: 0

Accepted Answers: (Type: Range) 52.8,54.8

0.25 points

- 12) Continue the process again and report the value of  $T(2)$  below

No, the answer is incorrect. Score: 0

Accepted Answers: (Type: Range) 47.6,49.6

0.25 points

### Problem 4: RK-4 Classic Method

We will now solve the above problem using single application (i.e., with  $h = 2$ ) of the Classical RK-4 Method, which is given as:

$$y_{i+1} = y_i + \frac{h}{6}(k_1 + 2k_2 + 2k_3 + k_4), \text{ where}$$

$$k_1 = f(y_i, t_i) \quad k_2 = f\left(y_i + \frac{hk_1}{2}, t_i + \frac{h}{2}\right) \quad k_3 = f\left(y_i + \frac{hk_2}{2}, t_i + \frac{h}{2}\right) \quad k_4 = f(y_i + hk_3, t_i + h)$$

- 13) Compute the value of  $k_1$

No, the answer is incorrect. Score: 0

Accepted Answers: (Type: Range) -25.5,-24.5

0.2 points

- 14) Compute the value of  $k_2$

No, the answer is incorrect. Score: 0

Accepted Answers: (Type: Range) -13,-12

0.2 points

- 15) Compute the value of  $k_3$

No, the answer is incorrect. Score: 0

Accepted Answers: (Type: Range) -19.5,-18

0.2 points

- 16) Compute the value of  $k_4$

No, the answer is incorrect. Score: 0

Accepted Answers: (Type: Range) -6.5,-6

0.2 points

- 17) Hence compute the value of  $T(2)$

No, the answer is incorrect. Score: 0

Accepted Answers: (Type: Range) 47.75,49.75

0.2 points