Unit 11 - Week 9: Numerical Integration

Course outline

How to access the portal?

Course Pre-requisites and Introduction

Week 1 - Computation and Error Analysis

Week 2 - Linear Systems and Equations

Week 3 - Linear Equations - 2

Week 4: Nonlinear Equations in Single Variable

Week 5: Nonlinear equations in Single and Multiple Variables

Week 6: Regression (Curve Fitting)

Week 7: Interpolation

Week 8: Numerical Differentiation

Week 9: Numerical Integration

- Numerical Integration: Introduction
- Trapezoidal rule and Derivation
- Simpson's Rules for Integration
- Bonus: MS-Excel for Numerical Integration
- Error Analysis for Simpson's Rules
- Numerical Integration Examples
- Bonus: Integration using MS-Excel
- Summary of "Newton Cotes" Formulae
- Richardson's Extrapolation
- Gauss Quadrature
- Summary of Numerical Integration
- Quiz : Assignment 9
- Numerical Methods for Engineers : Week 9 Feedback form
- Solutions to Assignment-9

Week 10: Ordinary Differential Equations – Initial Value Problems (ODE-IVP)

Week 11: ODE-IVP (Part-2)

Week 12: ODE - Boundary Value Problems

Video Download, Live Session and Other Information

Info about our Final Exam

Assignment 9

The due date for submitting this assignment has passed. **Due on 2019-10-02, 23:59 IST.**
As per our records you have not submitted this assignment.

Problem 1: Numerical Integration: Newton Cotes Formula

Compute $\int_0^{\frac{\pi}{2}} xsin(\sqrt{x})dx$ using Newton-Cotes formulae.

1) Compute the integral using single application of Trapezoidal rule

No, the answer is incorrect.
Score: 0
Accepted Answers:
(Type: Range) 0.09,0.0915

0.2 points

2) Compute the integral using single application of Simpson's $1/3^{rd}$ rule

No, the answer is incorrect.
Score: 0
Accepted Answers:
(Type: Range) 0.0745,0.0755

0.2 points

3) If one were to use the same step-size as $1/3^{rd}$ rule, we will require two applications of Trapezoidal rule. Hence, for a fair comparison, please compute the integral with two applications of the trapezoidal rule (i.e., with $h = \pi/12$)

No, the answer is incorrect.
Score: 0
Accepted Answers:
(Type: Range) 0.0785,0.0795

0.2 points

4) Compute the integral using single application of Simpson's $3/8^{th}$ rule

No, the answer is incorrect.
Score: 0
Accepted Answers:
(Type: Range) 0.0743,0.0753

0.2 points

5) If one were to use the same step-size as $3/8^{th}$ rule, we will require three applications of Trapezoidal rule. Hence, for a fair comparison, please compute the integral with three applications of the trapezoidal rule (i.e., with $h = \pi/18$)

No, the answer is incorrect.
Score: 0
Accepted Answers:
(Type: Range) 0.076,0.077

0.2 points

Problem 2: Numerical Integration: Quadrature :

Now, lets compute the same integral using quadrature approach. To do so, you must rewrite the above integral in the form: $\int_{-1}^{1} g(\bar{x})d\bar{x}$

6) Compute using first-order Gauss Quadrature, where

$$\int_{-1}^{1} g(\bar{x})d\bar{x} = g\left(-\frac{1}{\sqrt{3}}\right) + g\left(\frac{1}{\sqrt{3}}\right)$$

No, the answer is incorrect.
Score: 0
Accepted Answers:
(Type: Range) 0.0737,0.0747

0.5 points

7) Compute using second-order Gauss Quadrature, where

$$\int_{-1}^{1} g(\bar{x})d\bar{x} = \frac{5}{9}g\left(-\sqrt{\frac{3}{5}}\right) + \frac{8}{9}g(0) + \frac{5}{9}g\left(\sqrt{\frac{3}{5}}\right)$$

No, the answer is incorrect.
Score: 0
Accepted Answers:
(Type: Range) 0.074,0.076

0.5 points

Problem 3: Distance traveled by a rocket

In previous assignment, we were given distance traveled by a rocket and we computed velocity and acceleration. In this problem, given the velocity, we will compute the distance travelled by the rocket. The velocity is given by: $v(t) = \begin{cases} 0.631t^{1.2}, & t \leq 0 \\ 8 + 0.2t, & 10 < t \leq 20 \\ 12 & 20 < t \leq 40 \end{cases}$

8) Compute the distance traveled by the rocket in 40 seconds using Trapezoidal rule with h = 5

No, the answer is incorrect.
Score: 0
Accepted Answers:
(Type: Range) 390,400

0.5 points

9) Compute the distance traveled by the rocket in 40 seconds using Trapezoidal rule with h = 1

No, the answer is incorrect.
Score: 0
Accepted Answers:
(Type: Range) 390,400

0.5 points

Problem 4: Calculation of the amount of nutrients delivered

A nutrient is administered by diluting it with water. The flowrate, Q (ml/min) and the nutrient concentration C (µg/ml) both vary with time. The total amount of nutrient delivered in one hour is:

$$m = \int_0^{60} Q(t)C(t)dt$$

The following data is given:

t (min)	0	10	20	30	40	50	60
Q (ml/min)	52	45	48	46	53	50	47
C (µg/ml)	1.2	1.5	2.4	1.9	2.0	2.2	1.6

10) Use trapezoidal rule to compute the amount of nutrient delivered, in µg

No, the answer is incorrect.
Score: 0
Accepted Answers:
(Type: Range) 5540,5560

0.25 points

11) We will now use Trapezoidal rule with step-size of 20. In other words, use only the data at times 0, 20, 40 and 60 to compute the value of the integral

No, the answer is incorrect.
Score: 0
Accepted Answers:
(Type: Range) 5790,5810

0.25 points

12) Richardson's Extrapolation: We will now use the above two results to compute a better value using Richardson's extrapolation. To do so, we note that the global truncation error in Trapezoidal rule is $\mathcal{O}(h^2)$. Hence, an improved integral value is given by:

$$I = \frac{2^2 I_{(h=10)} - I_{(h=20)}}{2^2 - 1}$$

No, the answer is incorrect.
Score: 0
Accepted Answers:
(Type: Range) 5455,5475

0.25 points

13) Use Simpson's $1/3^{rd}$ rule to compute the amount of nutrient delivered, in µg

No, the answer is incorrect.
Score: 0
Accepted Answers:
(Type: Range) 5455,5475

0.25 points