Unit 10 - Week 8: Numerical Differentiation

Assignment 8

The due date for submitting this assignment has passed.
As per our records you have not submitted this assignment.

Due on 2019-09-25, 23:59 IST.

General Instructions

Please report all the results accurate to three or four significant digits. Numerical differentiation is very susceptible to round-off errors. So, please retain more digits in your calculations for reasonable accuracy.

Problem 1: Numerical Derivatives

Let: $y = x\sin(\sqrt{x})$

With step size of $h = 0.05$, compute y' and y'' at $x = \pi/6$

Note: (i) In trigonometric functions, we will only use radians; (ii) Use π/6 in your calculator, without rounding it off for appropriate calculation.

1) Please report $y'(\pi/6)$ using central difference formula

No, the answer is incorrect.
Score: 0
Accepted Answers:
(Type: Range) 0.923,0.943

0.25 points

2) Also report the value of $y'(\pi/6)$ using forward difference formula

No, the answer is incorrect.
Score: 0
Accepted Answers:
(Type: Range) 0.838,0.958

0.25 points

3) Also report the value of $y'(\pi/6)$ using backward difference formula

No, the answer is incorrect.
Score: 0
Accepted Answers:
(Type: Range) 0.905,0.927

0.25 points

4) Please report $y''(\pi/6)$ using central difference formula

No, the answer is incorrect.
Score: 0
Accepted Answers:
(Type: Range) 0.599,0.619

0.25 points

Problem 2: Error in Numerical Derivatives

In the above problem, the true value of the derivative is $\frac{dy}{dx} = x\sin(\sqrt{x}) + \frac{\sqrt{x}}{2}\cos(\sqrt{x})$

5) In the previous problem, you computed $y'(\pi/6)$ using forward difference formula. What is the absolute error between the numerical derivative and the true value?
(Please report the result accurate to at least three significant digits. E.g., if the answer is 0.0012345, please report 0.00123)

No, the answer is incorrect.
Score: 0
Accepted Answers:
(Type: Range) 0.0145,0.0149

0.1 points

6) Compute $y'(\pi/6)$ using forward difference formula, but with a lower step-size of h = 0.025. Report the absolute value of error of the numerical derivative.

No, the answer is incorrect.
Score: 0
Accepted Answers:
(Type: Range) 0.0068,0.0076

0.2 points

7) Based on the above two results, what is the order of accuracy of the forward difference formula? Note that the absolute error will: **0.2 points**
Decrease by a factor of 1/2 for an $\mathcal{O}(h^1)$ method
Decrease by a factor of 1/4 for an $\mathcal{O}(h^2)$ method
Decrease by a factor of 1/8 for an $\mathcal{O}(h^3)$ method

- Forward difference is an $\mathcal{O}(h^1)$ method
- Forward difference is an $\mathcal{O}(h^2)$ method
- Forward difference is an $\mathcal{O}(h^3)$ method
- Forward difference is an $\mathcal{O}(h^4)$ method

No, the answer is incorrect.
Score: 0
Accepted Answers:
Forward difference is an $\mathcal{O}(h^1)$ method

8) We will repeat the same procedure for central difference formula. You calculated y' in Problem-1. Please report the absolute value of error in calculation of y' using central difference formula

No, the answer is incorrect.
Score: 0
Accepted Answers:
(Type: Range) 5.2e-4,6e-4

0.1 points

9) Now compute $y'(\pi/6)$ using central difference formula, but with a lower step-size of h = 0.025. Report the absolute value of error in the numerical derivative.

No, the answer is incorrect.
Score: 0
Accepted Answers:
(Type: Range) 1.4e-04,2e-04

0.2 points

10) Based on the above two results, what is the order of accuracy of the central difference formula? **0.2 points**

- Central difference is an $\mathcal{O}(h^1)$ method
- Central difference is an $\mathcal{O}(h^2)$ method
- Central difference is an $\mathcal{O}(h^3)$ method
- Central difference is an $\mathcal{O}(h^4)$ method

No, the answer is incorrect.
Score: 0
Accepted Answers:
Central difference is an $\mathcal{O}(h^2)$ method

Problem 3: Computation of the rocket velocity and acceleration

Let us consider a problem of computing velocity and acceleration for a rocket. This is modified from an exercise problem in the *Numerical Methods* book by Chapra and Canale. The data for distance traveled by a rocket is given below:

Time, t (s)	0	10	20	30	40	50
Distance, z (km)	0	11	24	39	55	72

Velocity is given by $v = dz/dt$ and acceleration is $a = d^2z/dt^2$

11) Use central difference formula to compute velocity (in kg/s) at t = 10.

No, the answer is incorrect.
Score: 0
Accepted Answers:
(Type: Range) 1.19,1.21

0.25 points

12) Use central difference formula to compute acceleration (in kg/s2) at t = 10.

No, the answer is incorrect.
Score: 0
Accepted Answers:
(Type: Range) 0.019,0.021

0.25 points

13) Use central difference formula to compute velocity (in kg/s) at t = 30.

No, the answer is incorrect.
Score: 0
Accepted Answers:
(Type: Range) 1.54,1.56

0.25 points

14) Use central difference formula to compute acceleration (in kg/s2) at t = 30

No, the answer is incorrect.
Score: 0
Accepted Answers:
(Type: Range) 0.0099,0.011

0.25 points

Problem 4: Newton's Law of Cooling

Forced convection is used to cool a body faster. A hot metal ball was kept for rapid cooling under a current of air at $T_a = 25$ deg C. Newton's law of cooling is given by:
$\frac{dT}{dt} = -\beta(T - T_a)$
Following temperature vs. time data is obtained from experiments:

Time, t (s)	0	10	20	30	40	50
Temperature, T (deg C)	100	71	52	42	35	31

We will first compute dT/dt, and hence compute β

Repeat the above process to compute dT/dt at t = 10, 20, 30 and 40

15) Use central difference formula to dT/dt at t = 10

No, the answer is incorrect.
Score: 0
Accepted Answers:
(Type: Range) -2.42,-2.38

0.25 points

16) You now know the right-hand side of the above equation at t = 10. Use the value of temperature of the body and air temperature to compute β

No, the answer is incorrect.
Score: 0
Accepted Answers:
(Type: Range) 0.052,0.0525

0.25 points

17) Use central difference formula to dT/dt at t = 20

No, the answer is incorrect.
Score: 0
Accepted Answers:
(Type: Range) -1.47,-1.43

0.25 points

18) Use the above value and that of the temperatures to compute β at t = 20

No, the answer is incorrect.
Score: 0
Accepted Answers:
(Type: Range) 0.0535,0.054

0.25 points

Problem 5: Newton's Law of Cooling: Curve Fitting

In the above problem, you computed dT/dt at t = 10, 20, 30 and 40. Let us represent $Y = dT/dt$. Let us set up this problem by listing the data in the form below:

t	10	20	30	40
T	71	52	42	35
Y	____	____	____	____

Use this table above that you just created and fit a straight line of the form: $Y = a_0 + a_1 T$. You will be able to calculate the value of β from a_1 and the value of T_a from a_0 and a_1.

19) Please report the value of β calculated here

No, the answer is incorrect.
Score: 0
Accepted Answers:
(Type: Range) 0.0515,0.0525

0.5 points

20) Please report the updated value of T_a calculated here. (Please note that if you got the answer as 25, you perhaps did not follow the instructions correctly!)

No, the answer is incorrect.
Score: 0
Accepted Answers:
(Type: Range) 24.6,24.99

0.5 points