Unit 7 - Week 5: Nonlinear equations in Single and Multiple Variables

Assignment 5

The due date for submitting this assignment has passed.
As per our records you have not submitted this assignment.

Due on 2019-09-04, 23:59 IST.

In the following three problems, we will consider model of a continuous stirred reactor where a chemical reaction $A \rightarrow B$ takes place. The following model describes the system

$$\frac{C_{i0} - C_A}{\tau} - \frac{k\sqrt{C_A}}{K + C_A} = 0$$

where, $k = 1, K = 0.25, C_{i0} = 1, \tau = 0.25$. Note that C_A is the only unknown quantity.You will solve the above nonlinear equation using Newton-Raphson and Fixed-Point Iteration methods in the problems below.

Problem 1: Newton-Raphson

Starting with initial guesses of $C_A^{(0)} = 0.5$, use the Newton Raphson method to obtain the solution C_A. Please report the results as described below, accurate to three digits after the decimal. So, if the answer is 1.23456, then please report either 1.234 or 1.235

1) Please report the value of $f(C_A)$ used for the first iteration

No, the answer is incorrect.
Score: 0
Accepted Answers:
(Type: Range) 1.054,1.059

0.25 points

2) Please report the value of $f'(C_A)$ used for the first iteration

No, the answer is incorrect.
Score: 0
Accepted Answers:
(Type: Range) -3.70,-3.67

0.25 points

3) Hence compute and report the value of $C_A^{(1)}$ after the first iteration

No, the answer is incorrect.
Score: 0
Accepted Answers:
(Type: Range) 0.785,0.787

0.25 points

4) Please repeat once again, and report the value $C_A^{(2)}$ after the second iteration

No, the answer is incorrect.
Score: 0
Accepted Answers:
(Type: Range) 0.785,0.787

0.25 points

Problem 2: Fixed-Point Iteration

Part 1:

In this problem, we rearrange the nonlinear equation above as:

$$C_A = C_{i0} - \tau R_A$$

Starting with an initial guess of $C_A^{(0)} = 0.5$, please use fixed-point iteration and report the following results accurate to three digits after the decimal

5) Please report the value after the first iteration, i.e., $C_A^{(1)}$

No, the answer is incorrect.
Score: 0
Accepted Answers:
(Type: Range) 0.763,0.765

0.25 points

6) Please report the value after the second iteration, i.e., $C_A^{(2)}$

No, the answer is incorrect.
Score: 0
Accepted Answers:
(Type: Range) 0.784,0.785

0.25 points

7) Please report the value after the tenth iteration, i.e., $C_A^{(10)}$

No, the answer is incorrect.
Score: 0
Accepted Answers:
(Type: Range) 0.7859,0.787

0.25 points

Part 2 :

In this case, we let us rearrange the equation as:

$$C_A = \left(\frac{C_{i0} - C_A}{\tau} \cdot \frac{K + C_A}{k}\right)^2$$

Use the initial guess $C_A^{(0)} = 0.5$. Please report the results accurate to three significant digits.

If you run this for a couple more iterations, you will notice that fixed-point iteration diverges. The reason for this behavior was discussed in the videos on stability of fixed-point iteration

8) Please report the value after the first iteration, i.e., $C_A^{(1)}$

No, the answer is incorrect.
Score: 0
Accepted Answers:
(Type: Range) 2.23,2.27

0.25 points

Problem 3: Newton-Raphson in Two Variable

We have following two non-linear equation in two variables:

$$x_1^2 + 2x_1x_2 - x_2 - 10 = 0$$
$$x_1^2 + x_1x_2 - x_1 - 6 = 0$$

Starting with initial guesses of $x_1^{(0)} = 5$ and $x_2^{(0)} = 6$, we will use Newton Raphson method to find the solution. In the questions below, please report the results at various steps. Please report results accurate to three significant digits

9) First, compute the vector $f(x)$. Report the first element of the vector below.

No, the answer is incorrect.
Score: 0
Accepted Answers:
(Type: Range) 68.9,69.1

0.15 points

10) Report the second element of the vector $(f(x)$ below

No, the answer is incorrect.
Score: 0
Accepted Answers:
(Type: Range) 54.9,55.1

0.15 points

11) Next, calculate the Jacobian. The resulting Jacobian will be in the form

$$J = \begin{bmatrix} a_1 & a_2 \\ a_3 & 17 \end{bmatrix}$$

Please report a_1, i.e., the (1,1) element of the Jacobian

Hint

No, the answer is incorrect.
Score: 0
Accepted Answers:
(Type: Range) 21.9,22.1

0.15 points

12) Please report a_2, i.e., the (1,2) element of the Jacobian

No, the answer is incorrect.
Score: 0
Accepted Answers:
(Type: Range) 8.9,9.1

0.15 points

13) Please report a_3, i.e., the (2,1) element of the Jacobian

No, the answer is incorrect.
Score: 0
Accepted Answers:
(Type: Range) 4.9,5.1

0.15 points

14) Hence compute the first iterated value, $x^{(1)}$. Note that this is given by

$$x^{(1)} = x^{(0)} - J^{-1}f(x^{(1)})$$

Please report the first element of the vector $x^{(1)}$

No, the answer is incorrect.
Score: 0
Accepted Answers:
(Type: Range) 2.9,3.0

0.15 points

15) Please report the second element of the vector $x^{(1)}$.

No, the answer is incorrect.
Score: 0
Accepted Answers:
(Type: Range) 3.3,3.4

0.15 points

16) Repeat the entire process for one more iteration; and report the first element of the vector $x^{(2)}$

No, the answer is incorrect.
Score: 0
Accepted Answers:
(Type: Range) 2.1,2.2

0.15 points

17) Please report the second element of the vector $x^{(2)}$

No, the answer is incorrect.
Score: 0
Accepted Answers:
(Type: Range) 2.2,2.3

0.15 points

Problem 4: Dissolved Oxygen in a Lake (Bisection method)

Amount of dissolved oxygen in a lake gives an estimate of its health. The saturation concentration of dissolved oxygen in freshwater can be calculated using:

$$\ln(O_s) = -139.34411 + \frac{1.575701 \times 10^5}{T} - \frac{6.642308 \times 10^7}{T^2} + \frac{1.243800 \times 10^{10}}{T^3} - \frac{8.621949 \times 10^{11}}{T^4}$$

In the above equation, T is the temperature in Kelvins and O_s is the saturation oxygen concentration. The typical saturation concentrations ranges from 14.621 mg/L at 273 K to 6.413 mg/L at 313 K

The aim of this problem is to use bisection method to find temperature of fresh water that gives $O_s = 8.0mg/L$ using initial guesses as 273 K and 313 K

18) If the two initial guesses are $x^{(0)} = 273$ and $x^{(u)} = 313$, then estimate the number of iterations required by bisection method to converge to a solution with an error tolerance of 0.1. (Note: Number of iterations is an integer)

No, the answer is incorrect.
Score: 0
Accepted Answers:
(Type: Numeric) 9

0.25 points

19) What is the value after first iteration of bisection method, i.e., $x^{(1)}$?

No, the answer is incorrect.
Score: 0
Accepted Answers:
(Type: Range) 292.9,293.1

0.25 points

20) What is the value after the second iteration of bisection method, i.e., $x^{(2)}$?

No, the answer is incorrect.
Score: 0
Accepted Answers:
(Type: Range) 302.9,303.1

0.25 points

21) What is the converged value of the solution? Please report the temperature that gives $O_s = 8$, accurate to one digit after the decimal, using bisection rule

No, the answer is incorrect.
Score: 0
Accepted Answers:
(Type: Range) 299.5,300.5

0.25 points