Assignment 5

Unit 7 - Week 5: Nonlinear equations in Single and Multiple Variables

Problem 1: Newton-Raphson

1. Describe the class of problems that can be solved using the Newton-Raphson method. How is it used to find the roots of a nonlinear equation f(x) = 0? 
2. Explain the Newton-Raphson algorithm step by step. 
3. What is the role of the derivative f'(x) in the Newton-Raphson method? 
4. How can the method be adapted for finding multiple roots? 
5. Discuss the convergence properties of the Newton-Raphson method. 
6. Provide an example of a problem where the Newton-Raphson method would be particularly effective. 
7. Explain why the method may fail to converge or converge slowly. How can these issues be addressed? 

Problem 2: Fixed-Point Iteration

1. Define the fixed-point iteration method and explain its application to solving nonlinear equations. 
2. Describe the iterative process in detail. 
3. What is the significance of the fixed point in the fixed-point iteration method? 
4. Discuss the convergence criteria for the fixed-point iteration method. 
5. Provide an example of a problem that can be solved using the fixed-point iteration method. 
6. How does the choice of the initial guess affect the convergence of the fixed-point iteration method? 
7. Explain why the fixed-point iteration method may fail to converge or converge slowly. How can these issues be addressed? 

Problem 3: Newton-Raphson in Two-Variable

1. Explain how the Newton-Raphson method can be extended to solve systems of nonlinear equations. 
3. How is the solution for each variable obtained in a system of equations using the Newton-Raphson method? 
4. Discuss the convergence properties of the Newton-Raphson method for systems of equations. 
5. Provide an example of a problem where the Newton-Raphson method would be particularly effective for systems of equations. 
6. How does the method handle failure to converge or slow convergence in systems of equations? 
7. Explain why the method may fail to converge or converge slowly in systems of equations. How can these issues be addressed? 

Problem 4: Dichotomous Search in a Lake (Bisection method)

1. Explain the dichotomous search method and its application to finding roots of nonlinear equations. 
2. Describe the iterative process used in the dichotomous search method. 
3. Explain how the method determines the root of a function f(x) = 0. 
4. Discuss the convergence criteria for the dichotomous search method. 
5. Provide an example of a problem that can be solved using the dichotomous search method. 
6. How does the choice of the initial interval affect the convergence of the dichotomous search method? 
7. Explain why the dichotomous search method may fail to converge or converge slowly. How can these issues be addressed?