Unit 5 - Week 3 - Linear Equations - 2

Assignment 3

Problem 1: Gauss-Seidel: First Iteration

Given a system of linear equations:

\[ \begin{align*}
2x + 3y &= 15 \\
3x + 4y &= 20
\end{align*} \]

Find the approximate values of \( x \) and \( y \) using the Gauss-Seidel method.

- \( x_{0} = 0 \), \( y_{0} = 0 \)
- \( x_{1} = \frac{1}{2}(15 - 3y_{0}) \)
- \( y_{1} = \frac{1}{3}(20 - 4x_{0}) \)

\[ \begin{align*}
x_{1} &= \frac{1}{2}(15 - 3(0)) = 7.5 \\
y_{1} &= \frac{1}{3}(20 - 4(0)) = 6.67
\end{align*} \]

Problem 2: Gauss-Seidel: Fifth Iteration

Given the system of linear equations:

\[ \begin{align*}
5x + 2y &= 17 \\
2x + 4y &= 23
\end{align*} \]

Find the approximate values of \( x \) and \( y \) using the Gauss-Seidel method after five iterations.

- \( x_{0} = 0 \), \( y_{0} = 0 \)
- \( x_{1} = \frac{1}{5}(17 - 2y_{0}) \)
- \( y_{1} = \frac{1}{4}(23 - 2x_{0}) \)

Repeat for five iterations:

<table>
<thead>
<tr>
<th>Iteration</th>
<th>( x )</th>
<th>( y )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>3.4</td>
<td>5.5</td>
</tr>
<tr>
<td>2</td>
<td>3.2</td>
<td>5.7</td>
</tr>
<tr>
<td>3</td>
<td>3.1</td>
<td>5.9</td>
</tr>
<tr>
<td>4</td>
<td>3.0</td>
<td>6.1</td>
</tr>
<tr>
<td>5</td>
<td>2.9</td>
<td>6.3</td>
</tr>
</tbody>
</table>

Problem 3: Jacobi Iteration

Given the system of linear equations:

\[ \begin{align*}
3x + 2y &= 16 \\
2x + 4y &= 24
\end{align*} \]

Find the approximate values of \( x \) and \( y \) using the Jacobi method after five iterations.

- \( x_{0} = 0 \), \( y_{0} = 0 \)
- \( x_{1} = \frac{1}{3}(16 - 2y_{0}) \)
- \( y_{1} = \frac{1}{4}(24 - 2x_{0}) \)

Repeat for five iterations:

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</tr>
<tr>
<td>4</td>
<td>4.9</td>
<td>3.6</td>
</tr>
<tr>
<td>5</td>
<td>4.8</td>
<td>3.8</td>
</tr>
</tbody>
</table>

Problem 4: Tri-Diagonal Matrix Algorithm

Given a system of linear equations in tridiagonal form:

\[ \begin{align*}
3x_{1} - x_{2} &= 4 \\
-x_{1} + 4x_{2} &= 6 \\
-2x_{2} + 4x_{3} &= 8
\end{align*} \]

Find the solution using the Tri-Diagonal Matrix Algorithm.

- \( x_{0} = 0 \), \( x_{1} = 0 \), \( x_{2} = 0 \)
- \( x_{1} = \frac{1}{3}(4 + x_{2}) \)
- \( x_{2} = \frac{1}{4}(6 + x_{3}) \)
- \( x_{3} = \frac{1}{4}(8 + 0) \)

\[ \begin{align*}
x_{1} &= \frac{1}{3}(4 + 0) = 1.33 \\
x_{2} &= \frac{1}{4}(6 + 2) = 2.5 \\
x_{3} &= \frac{1}{4}(8) = 2
\end{align*} \]

Problem 5: Building an Application Problem

Create a real-world problem that can be solved using the Gauss-Seidel method. Describe the problem, the equations involved, and the solution process.

Example: A company needs to optimize its production schedule. They have three machines and a limited number of tasks to complete. Each machine can perform a certain number of tasks per day. Using the Gauss-Seidel method, determine the optimal schedule to maximize production.

\[ \begin{align*}
\text{Machine 1} & : 2x_{1} + y_{1} = 10 \\
\text{Machine 2} & : x_{2} + 2y_{2} = 15 \\
\text{Machine 3} & : x_{3} + y_{3} = 20
\end{align*} \]

Use the Gauss-Seidel method to find the optimal number of tasks for each machine.