

# Unit 4 - Week 2 - Linear Systems and Equations

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## Assignment 2

The due date for submitting this assignment has passed. **Due on 2019-08-21, 23:59 IST.**  
 As per our records you have not submitted this assignment.

Consider the following example of linear equations in three variables

$$\begin{aligned} 5x_1 + 4x_3 + x_4 &= -6 \\ 4x_1 + 5x_2 + 2x_3 &= 8 \\ 4x_1 + 4x_2 + 4x_3 + x_4 &= 1 \\ x_1 + 2x_2 + 2x_3 + 3x_4 &= 2 \end{aligned}$$

Please write the equations in the form  $Ax = b$ , and solve the following three problems based on this example.

### Problem 1: Gauss Elimination (First operation)

In the first problem, you will report the results of performing Gauss Elimination (without partial pivoting). To do so, define a matrix  $Ab = [A | b]$ , obtained by concatenating matrix A with vector b. Perform one sequence of Gauss Elimination to obtain zeros in the first pivot column. The resulting matrix will be

$$Ab = \begin{bmatrix} * & * & * & * & * \\ 0 & \bar{a}_{21} & * & * & \bar{\beta}_2 \\ 0 & * & * & \bar{a}_{34} & * \\ 0 & * & * & * & \bar{\beta}_4 \end{bmatrix}$$

1) The value of element  $\bar{\beta}_2 =$  \_\_\_\_\_ (i.e., second row of modified "b" vector)

No, the answer is incorrect. Score: 0

Accepted Answers: (Type: Range) 12.79,12.81

0.25 points

2) The value of element  $\bar{\beta}_4 =$  \_\_\_\_\_ (i.e., last row of modified "b" vector)

No, the answer is incorrect. Score: 0

Accepted Answers: (Type: Range) 3.19,3.21

0.25 points

3) The value of element  $\bar{a}_{21} =$  \_\_\_\_\_

No, the answer is incorrect. Score: 0

Accepted Answers: (Type: Range) 4.99,5.01

0.25 points

4) The value of element  $\bar{a}_{34} =$  \_\_\_\_\_

No, the answer is incorrect. Score: 0

Accepted Answers: (Type: Range) 0.19,0.21

0.25 points

### Problem 2: Gauss Elimination (Second operation)

Another step of Gauss Elimination was run to obtain:

$$Ab = \begin{bmatrix} * & * & * & * & * \\ 0 & * & * & * & * \\ 0 & 0 & * & * & * \\ 0 & 0 & * & \bar{a}_{44} & \bar{\beta}_4 \end{bmatrix}$$

5) The value of element  $\bar{a}_{44} =$  \_\_\_\_\_? Please report the result accurate to three significant digits.

No, the answer is incorrect. Score: 0

Accepted Answers: (Type: Range) 3.11,3.13

0.25 points

6) The value of element  $\bar{\beta}_4 =$  \_\_\_\_\_? Please report the result accurate to three significant digits.

No, the answer is incorrect. Score: 0

Accepted Answers: (Type: Range) -1.93,-1.91

0.25 points

Thereafter, complete the Gauss Elimination to obtain lower-triangular matrix of the form

$$Ab = \begin{bmatrix} * & * & * & * & * \\ 0 & * & * & * & * \\ 0 & 0 & * & * & * \\ 0 & 0 & 0 & \bar{a}_{44} & \bar{\beta}_4 \end{bmatrix}$$

7) The value of element  $\bar{a}_{44} =$  \_\_\_\_\_? Please report the result accurate to three significant digits.

No, the answer is incorrect. Score: 0

Accepted Answers: (Type: Range) 2.30,2.33

0.25 points

8) The value of element  $\bar{\beta}_4 =$  \_\_\_\_\_? Please report the result accurate to three significant digits.

No, the answer is incorrect. Score: 0

Accepted Answers: (Type: Range) 2.30,2.33

0.25 points

### Problem 3: LU Decomposition

In the lectures, we saw that the result of Gauss Elimination can be used for LU decomposition. For the Matrix A in the above example, perform the LU decomposition. The Matrix obtained after Gauss Elimination in Problem-2 is the U matrix. The L matrix is given by the factors determined during Gauss

Elimination. Let  $L = \begin{bmatrix} d & 0 & 0 & 0 \\ \alpha_{21} & d & 0 & 0 \\ \alpha_{31} & \alpha_{32} & d & 0 \\ \alpha_{41} & \alpha_{42} & \alpha_{43} & d \end{bmatrix}$

9) The value of diagonal elements of L matrix is d = \_\_\_\_\_?

 0  
 1  
 Some other value

No, the answer is incorrect. Score: 0

Accepted Answers: 1

0.25 points

10) The value of element  $\alpha_{21} =$  \_\_\_\_\_?

No, the answer is incorrect. Score: 0

Accepted Answers: (Type: Range) 0.79,0.81

0.25 points

11) The value of element  $\alpha_{31} =$  \_\_\_\_\_?

No, the answer is incorrect. Score: 0

Accepted Answers: (Type: Range) 0.79,0.81

0.25 points

12) The value of element  $\alpha_{42} =$  \_\_\_\_\_?

No, the answer is incorrect. Score: 0

Accepted Answers: (Type: Range) 0.39,0.41

0.25 points

### Problem 4: Back-substitution

Let us consider another example of an Ab matrix obtained after Gauss Elimination. (Please note I have used a different example to obtain this matrix). Please use Back-Substitution to obtain the solution to the linear equations. The Ab matrix is given by

$$Ab = \begin{bmatrix} 5 & 0 & 4 & 1 & -6 \\ 0 & 6.25 & -1.5 & -1 & 16 \\ 0 & 0 & 2 & 1 & -5 \\ 0 & 0 & 0 & 2 & 2 \end{bmatrix}$$

13) The value of the first element of the solution vector,  $x_1 =$  \_\_\_\_\_? (Please note that  $x_1$  is the last value obtained in back-substitution)

No, the answer is incorrect. Score: 0

Accepted Answers: (Type: Range) 0.95,1.05

0.25 points

14) The value of  $x_2 =$  \_\_\_\_\_?

No, the answer is incorrect. Score: 0

Accepted Answers: (Type: Range) 1.95,2.05

0.25 points

15) The value of  $x_3 =$  \_\_\_\_\_?

No, the answer is incorrect. Score: 0

Accepted Answers: (Type: Range) -3.05,-2.95

0.25 points

16) The value of  $x_4 =$  \_\_\_\_\_? (Please note that  $x_4$  is the first value obtained in back-substitution, since we start from the end and calculate backwards)

No, the answer is incorrect. Score: 0

Accepted Answers: (Type: Range) 0.95,1.05

0.25 points

### Problem 5: Application Problem

A Farm prepares a special feed, which is a mixture of corn, horse-gram and peas. Corn cost ₹ 40 per kg, Horse-gram cost ₹ 50 per kg and peas cost ₹ 90 per kg. The special feed is made such that the weight of corn is twice that of horse-gram. Please answer the following questions

17) How many kilograms of peas should be used to produce 140 kg of the feed at ₹60 per kg?

No, the answer is incorrect. Score: 0

Accepted Answers: (Type: Range) 49.9,50.1

0.5 points

18) How many kilograms of peas should be used to produce 100 kg of the feed at ₹76 per kg?

No, the answer is incorrect. Score: 0

Accepted Answers: (Type: Range) 69.9,70.1

0.5 points