Week 7 Assignment 1

Exercise 1 (cont.):

a. If Y is a random variable, then

\[ E(Y) = \sum y \cdot f(y) \]

b. The expected value of a constant k is k.

c. The expected value of a random variable Y is a constant k.

Exercise 2:

Which of the following is a valid pdf for x > 0?

\[ f(x) = \begin{cases} \frac{1}{x^2} & \text{if } x > 1 \\ 0 & \text{otherwise} \end{cases} \]

Exercise 3:

Given the function

\[ f(x) = \begin{cases} 2x & \text{if } 0 < x < 1 \\ 0 & \text{otherwise} \end{cases} \]

Find the constant c.

Exercise 4:

Which of the following is a valid pdf for x > 0?

\[ f(x) = \begin{cases} \frac{1}{x^2} & \text{if } x > 1 \\ 0 & \text{otherwise} \end{cases} \]

Exercise 5:

Define two random variables X and Y. The joint pdf of X and Y is given by

\[ f(x, y) = \begin{cases} cx & \text{if } 0 < x < 1, 0 < y < 1 \\ 0 & \text{otherwise} \end{cases} \]

Find the value of c.

Exercise 6:

Let X be a random variable with pdf f(x) = \begin{cases} e^{-x} & \text{if } x > 0 \\ 0 & \text{otherwise} \end{cases}

Find the cumulative distribution function (CDF) of X.

Exercise 7:

The variance of the random variable X is given by

\[ \text{Var}(X) = E(X^2) - [E(X)]^2 \]

Exercise 8:

The random variable X is normally distributed with mean \( \mu \) and standard deviation \( \sigma \). The probability density function (PDF) of X is given by

\[ f(x) = \frac{1}{\sqrt{2\pi \sigma^2}} e^{-\frac{(x-\mu)^2}{2\sigma^2}} \]

Exercise 9:

A job shop has four random variables: X1, X2, X3, and X4. The joint probability density function (PDF) of these variables is given by

\[ f(x_1, x_2, x_3, x_4) = \begin{cases} \frac{1}{4} & \text{if } 0 < x_1 < 1, 0 < x_2 < 1, 0 < x_3 < 1, 0 < x_4 < 1 \\ 0 & \text{otherwise} \end{cases} \]

Find the probability that all four variables are between 0 and 1.