Assignment 7

Due on 2023-06-16, 23:59 IST.

Consider a first order IIR filter 

\[ H(z) = \frac{1 - a}{1 - az^{-1}} \]

for \( |a| < 1 \).

(a) \( |H(e^{j\omega})| \) is constant and \( \angle H(e^{j\omega}) \) is linear in \( \omega \) for all \( \omega \) in the range from \( -\pi \) to \( \pi \).

(b) \( |H(e^{j\omega})| \) is constant but \( \angle H(e^{j\omega}) \) is not linear in \( \omega \) for all \( \omega \) in the range from \( -\pi \) to \( \pi \).

(c) \( |H(e^{j\omega})| \) is variable and \( \angle H(e^{j\omega}) \) is linear in \( \omega \) for all \( \omega \) in the range from \( -\pi \) to \( \pi \).

(d) \( |H(e^{j\omega})| \) is variable but \( \angle H(e^{j\omega}) \) is not linear in \( \omega \) for all \( \omega \) in the range from \( -\pi \) to \( \pi \).

A causal and stable low pass filter has impulse response \( h(n) \) and transfer function 

\[ H(z) = \frac{1}{(1 - \frac{1}{2}z^{-1})(1 - \frac{3}{4}z^{-1} + \frac{1}{4}z^{-2})} \].

A new filter is generated having impulse response \( h_1(n) = (-1)^n h(n) \). The transfer function of the filter is,

(a) \( H(z) = \frac{1 - \frac{1}{2}z^{-1}}{(1 + \frac{3}{4}z^{-1} - \frac{1}{4}z^{-1})} \).

(b) \( H(z) = \frac{1 - \frac{1}{2}z^{-1}}{(1 + \frac{1}{4}z^{-1} - \frac{1}{4}z^{-1})} \).

(c) \( H(z) = \frac{1 - \frac{1}{2}z^{-1}}{(1 + \frac{3}{4}z^{-1} - \frac{1}{4}z^{-1})} \).

(d) \( H(z) = \frac{1 - \frac{1}{2}z^{-1}}{(1 + \frac{3}{4}z^{-1} - \frac{1}{4}z^{-1})} \).

An IIR filter is designed from a prototype causal and stable analog filter \( h_a(n) = \frac{s^2 + 4s + 3}{s^2 + 3s + 1} \) by the impulse invariance method. In other words, if \( h_a(t) \) is the impulse response of the analog filter, then \( h_b(t) = T \cdot h_a(\frac{t}{T}) \) is the sampled version of \( h_a \). The IIR filter \( H(z) = \frac{N(z)}{D(z)} \) will then have \( |N(z)| = |D(z)| \).

(a) \( 1 + (e^{-T} - e^{-3T})e^{-t} \).

(b) \( e^{-T} - e^{-3T} \).

(c) \( e^{-3T} - e^{-T} \).

(d) \( 1 - e^{-T} - e^{-3T} \).

Given that \( H(z) \) is a causal and stable IIR filter, if \( z \) in \( H(z) \) is replaced by \(-z^2\), the resulting filter will be

(a) neither stable nor causal

(b) stable but non-causal

(c) unstable but causal

(d) both stable and causal

Given an analog filter \( H_a(s) = \frac{s^2 + 4s + 3}{s^2 + 3s + 1} \), and a digital filter \( H_d(z) \), designed from \( H_a(s) \) by the impulse invariance method, assuming sampling period \( T \). Then, \( H(z) \) is given by

(a) \( 1 - e^{-T}\sin(2T)z^{-1} \).

(b) \( 1 - e^{-T}\cos(2T)z^{-1} \).

(c) \( 1 - e^{-T}\sin(2T)z^{-1} \).

(d) \( 1 - e^{-T}\cos(2T)z^{-1} \).

Given an analog filter \( H_a(s) = \frac{s + 1}{s^2 + 2s + 1} \), and a digital filter \( H_d(z) \), designed from \( H_a(s) \) by the impulse invariance method, assuming sampling period \( T \). Then, \( H(z) \) is given by

(a) \( 1 - e^{-T}\sin(2T)z^{-1} \).

(b) \( 1 - e^{-T}\cos(2T)z^{-1} \).

(c) \( 1 - e^{-T}\sin(2T)z^{-1} \).

(d) \( 1 - e^{-T}\cos(2T)z^{-1} \).