Basic Tools on Microwave Engineering
Solution of Assignment-2: Impedance matching Using Smith Chart
HW 1: Design a lossless EL-section matching network for the following normalized load impedances.

a. $\bar{Z}_L = 1.5 - j2.0$

b. $\bar{Z}_L = 0.5 + j0.3$

Solution a.

Step-1:
- Determine the region. In this case the normalized load impedance lies in region 2.
- Let us choose the EL-section matching network $C_p - L_s$ or $L_s - C_p$ as shown below.

![Diagram of EL-section matching network]

Step-2:
- Plot the normalized load impedance, $\bar{Z}_L$ on the smith chart.
- Draw the corresponding constant VSWR circle.

Step-3: The next element to load is a reactive element added in shunt. Hence the conversion of impedance to its corresponding admittance is necessary.
- Convert $\bar{Z}_L$ to its corresponding admittance $\bar{Y}_L = 0.24 + j0.33$
- Identify the $\bar{G} = 1$ circle on the admittance smith chart.
- Move along the $\bar{G}_L = 0.24$ circle to reach the intersection of $\bar{G} = 1$ circle. Measure the new susceptance $j\bar{B}_1$ at that point.
Let it is $j0.42$ by going clockwise. So, the required susceptance to be added is $j\bar{B} = 0.42 - j0.33 = j0.09$. As, the $j\bar{B}$ is positive, it is a capacitor. Hence, $C_p = \frac{0.09}{\omega Z_0}$ F.

Alternatively one can go anticlockwise to reach $\bar{G} = 1$ circle and note new susceptance to be $j\bar{B}_2 = -0.42$. The required susceptance to be added is $j\bar{B}_2 = -j0.42 - j0.33 = -j0.75$. As, $j\bar{B}$ is negative, it is an inductor $L_p$. It’s value $L_p = \frac{Z_0}{0.75\omega}$ H.

**Step-4:** The next element to the shunt susceptance is a series reactance. Therefore conversion of admittance to impedance is needed.

- Convert the normalized admittance $\bar{Y}_1$ to its corresponding impedance $\bar{Z}_1$. The value is $\bar{Z}_1 = 1 - j1.8$

- Move along the $\bar{R} = 1$ circle clockwise to reach the origin. So, required reactance to be added is $+j1.8$. Therefore it is an inductor $L_s = \frac{1.8Z_0}{\omega}$ (H).

- Alternatively, convert $\bar{Y}_2$ to its corresponding impedance $\bar{Z}_2$. The value is $1 + j1.75$.

- Move along the $\bar{R} = 1$ circle to reach the origin.

So, the required reactance to be added is $-j1.75$ Hence, it is a capacitor with capacitance $C_p = \frac{1}{\omega Z_0}$.
Solution b.

Step-1:
- Plot $\tilde{Z}_L = 0.5 + j0.3$ on the smith chart and determine the region. In this case it is in region 1
- Let us choose the EL-section matching network as $L_s - C_p$ or $C_p - L_s$.

Step-2:
- Plot the normalized load impedance, $\tilde{Z}_L = 0.5 + j0.3$ on the smith chart.
Step-3:

- Identify the $\bar{G} = 1$ circle on the admittance smith chart.
- Move along the $\bar{R}_L = 0.5$ circle of impedance smith chart to reach the intersection of $\bar{G} = 1$ circle of admittance smith chart. Measure the new impedance $\bar{Z}_1$ at that point. So, $\bar{Z}_1 = 0.5 + j0.48$

So, the reactance to be added, $j\bar{X} = j0.48 - j0.3 = j0.18$. Hence the element is an inductor having inductance $L_s = \frac{0.18z_0}{\omega}$ H.

Step-4: The next element to the series reactance is a shunt susceptance. Therefore conversion of impedance to admittance is needed.

- Convert the normalized impedance $\bar{Z}_1$ to its corresponding admittance $\bar{Y}_1$, and for rest of the calculation assume the impedance smith chart as admittance chart.
- Note the value of $\bar{Y}_1$ to be $1 - j$.
- Add the required amount of Susceptance to reach the origin.

So, the susceptance to be added is $j1$. This is a capacitor $C_p$. Thus, $C_p = \frac{1}{z_0\omega}$ F. Alternatively one can determine the values of the series reactance and shunt susceptance to be $-j0.8$
and $-j1$ so, $C_s = \frac{1}{0.8Z_0\omega} F$ and $L_p = \frac{Z_0}{\omega} H$.

**HW 2:** Determine the value of inductor and capacitor used in the matching network described below at 500 MHz. Assume the characteristic impedance of the transmission line 50Ω.

Given $Z_L = 2 - j0.3$ and $Z_{in} = \frac{25+j15}{50} = 0.5 + j0.3$

**Step-1:** The next immediate element to load, in the matching network is a capacitor added in shunt. Therefore, it is required to convert the $Z_L$ to its corresponding admittance. So, the following operations are required to be performed.

- Plot the normalized load impedance $\tilde{Z}_L$ on the smith chart.
- Draw the constant VSWR circle.
- Convert the load impedance, $\tilde{Z}_L$ to its corresponding admittance, $\tilde{Y}_L$. The value of $\tilde{Y}_L$ is determined as $0.48 + j0.07$

**Step-2:** In this case the normalized input impedance $\tilde{Z}_{in}$ is not equal to its characteristic impedance. Therefore, to determine the capacitance value we need to find the $\tilde{G}_{in} = 0.5$ circle on admittance smith chart. So, the following operations are performed.

- Identify the $\tilde{G}_{in} = 0.5$ circle on the admittance smith chart.
- Move along the $\tilde{G}_L = 0.48$ circle on impedance smith chart to reach the closest intersection with $\tilde{G}_{in} = 0.5$ circle of admittance smith chart.
- Determine required value of susceptance by reading the value of the admittance, $\tilde{Y}_1$ on the intersection point.
- Draw the constant VSWR circle corresponding to $\tilde{Y}_1$
Step-3: The next element in the matching network is an inductor added in series. Therefore the following operations are required to be done

- Convert the admittance $\bar{Y}_1$ to its corresponding impedance $\bar{Z}_1$. If $R_1 = R_m$, then the matching is possible. Otherwise matching is not possible.
- Move along the $R_m = 0.5$ circle on impedance smith chart to reach the desired value of normalized input impedance.

In the entire process, we found that $B = 0.77$ and $X = 1.2$. So, the capacitance and inductance values can be calculated as 4.9 pF and 19.1 nH respectively.

HW 3: It is required to match a load impedance of $\bar{Z}_L = 112.5 + j102\Omega$ to $75\Omega$ line using a series short-circuit single stub. Determine the length of the stub and the distance at which the stub is required to be placed on the line, to obtain matching at 2 GHz

It is given that, $Z_L = 112.5 + j102\Omega$ and $Z_0 = 75\Omega$. Therefore, the normalized load impedance can be determined as $\bar{Z}_L = 1.5 + j1.4$. The following steps can be followed to determine the solution of the above problem.
**Step-1:**
- Plot the normalized load impedance $\bar{Z}_L$ on the smith chart.
- Draw a constant VSWR circle.

**Step-2:** We have to move toward the generator, along the constant VSWR circle to get the intersection point with the $\bar{R} = 1$ circle. So,
  - Identify the intersection point with $\bar{R} = 1$ circle and mark as $\bar{Z}_1$.
  - Measure the distance $d$, between $\bar{Z}_L$ and $\bar{Z}_1$. this will give the position at which the stub is required to be placed on the line.

**Step-3:** To obtain the length of the stub we have to do the following operations.
  - Move along the $\bar{R} = 1$ circle to reach the origin.
  - Determine the amount of reactance is required to obtain the matching.
  - Determine the length of the stub $l$. In this problem the stub is shorted. Therefore, the reference point would be $\bar{Z} = 0$. 
Hence by following the above procedure, the value of \( d \) and \( l \) are found equal as \( 0.14\lambda \). So, the physical length of the stub can be calculated as \( 0.14 \times \frac{3 \times 10^8}{2 \times 10^9} = 2.1\text{cm} \)

**HW 4:** Determine the length of the open circuit shunt-stubs and their corresponding susceptance values for the matching network given below. Assume the operating frequency to be 1 GHz.

In our problem it is given that \( Z_L = 60 + j25\Omega \) and \( Z_0 = 50\Omega \). So, the value of normalized impedance can be determined as \( \bar{Z}_L = 1.2+j0.5 \). It is also given that the frequency of operation is 1 GHz. Thus, the value of wavelength, \( \lambda \) can be determined as 30 cm. To obtain the solution of the above problem using smith chart, the following steps are required to be followed.

**Step-1:**

- Plot the normalized load impedance, \( \bar{Z}_L \) on the smith chart.
- Draw the VSWR circle corresponding to \( \bar{Z}_L \).
- Here the first stub is placed at a distance of 4.5 cm away, toward the generator. Hence, move along the constant VSWR circle towards the generator by an amount of \( 0.15\lambda \) (equivalent to 4.5 cm).
- Measure the new impedance \( \bar{Z}_1 \).

**Step-2:** as, the stubs are connected to the main line in shunt, hence, it is required to convert the normalized impedance \( \bar{Z}_1 \) to its corresponding admittance \( \bar{Y}_1 \). \( \bar{Y}_1 = 0.7 + j0.28 \)
• Convert the $Z_1$ to its corresponding admittance $Y_1$. For rest of the calculation assume the impedance smith chart as admittance smith chart.

• The stubs are separated by 3.75 cm, which is equivalent to $\frac{\lambda}{8}$, and 90° by angle. Now, rotate the $1 + jB$ circle by 90° towards the load.

• Move along the constant $\hat{G} = 0.7$ circle to reach the intersection with rotated $1 + jB$ circle.

• Measure the value of admittance $Y_2 = 0.7 + j1.95$ and draw the constant VSWR circle corresponding to $Y_2$.

• Determine the value of susceptance offered by the 1st stub: $jB_1 = 1.67$

**Step-3:**

• Move along the constant VSWR circle corresponding to $Y_2$ towards the generator by an amount of $\frac{\lambda}{8}$ to reach the $1 + jB$ circle.
Add the required susceptance to reach the origin. This will give us the susceptance value of the second stub. Hence, \( jB_2 = j2.58 \).

Step-4:

- From the measure values of susceptance of individual stubs, their corresponding lengths can be determined. So, plot the corresponding values of the susceptances on the smith chart.
- Move toward the generator from the reference point upto the corresponding susceptance values to determine the lengths. As, the stubs are of open circuit. Therefore, the reference point would be \( Y = 0 \).

By following the above procedure, the length of the first stub is determined as 0.164\(\lambda\). So the physical length would be 4.92 cm at 1 GHz. Similarly the length of second stub can be calculated as 0.186\(\lambda\) i.e 5.58 cm at 1 GHz.

**HW 5:** Design a single section quarter-wave transformer matching network to match a 100\(\Omega\) load to a 50\(\Omega\) transmission line \((f_0 = 1\ MHz)\). If atleast -0.18 dBm power is required to be delivered to the load when 0 dBm power is given in the input, then find out the percentage
bandwidth.

The length of the transformer is $\lambda/4 = 75m$ at 1 MHz. Now, the impedance required by the quarter-wave transformer can be determined by the formula given in Eq.(1) below:

$$Z_1 = \sqrt{Z_0 Z_L}$$  \(1\)

Therefore, $Z_1$ is determined as $\sqrt{(50)(100)} = 70.71\,\Omega$.

The maximum value of the reflection coefficient can be derived from the constraints imposed on the amount of delivered power to the load. By applying the concept of conservation of power, we can write that:

Transmitted Power + Reflected Power = Incident Power

antilog(-.18 dBm) + Reflected Power = antilog(0 dBm)

i.e. 0.96 mW + Reflected Power = 1 mW

As, the minimum amount of the delivered power to load is 0.96 mW therefore, the maximum amount of reflected power=0.04 mW

Therefore, $|\Gamma_{max}|^2 = \frac{\text{Reflected Power}}{\text{Incident Power}}$. Hence, $\Gamma_{max} = 0.2$.

Now, the percentage bandwidth for the specific VSWR tolerance can be obtained by using the formula given below.

$$\frac{\Delta f \times 100}{f_0} = 200 - \frac{400}{\pi} \cos^{-1} \left[ \frac{\Gamma_{max} \sqrt{Z_0 Z_L}}{\sqrt{1 - \Gamma_{max}^2}} \frac{2\sqrt{Z_0 Z_L}}{|Z_0 - Z_L|} \right]$$ \((2)\)

Hence, the percentage bandwidth is calculated as 75.2%.

**HW 6:** A load of $Z_L = 1.5$ is to be matched to a feed line using a multi-section transformer, and it is desired to have a passband response of $|\Gamma(\theta)| = A(0.1 + \cos^2(\theta))$ for $0 \leq \theta \leq \pi$. Design a two-section impedance matching network.

**Solution:**

Given, $N=2$ and

$$|\Gamma(\theta)| = A(0.1 + \cos^2(\theta)) = A(0.1 + \frac{1}{2} + \frac{\cos(2\theta)}{2})$$ \((3)\)

Now the generic equation is $\Gamma(\theta) = \Gamma_0 + \Gamma_1 e^{-j2\theta} + \Gamma_1 e^{-j4\theta}$

The above one can be approximated as: $\Gamma(\theta) = 2e^{-j2\theta}[\Gamma_0 \cos(2\theta) + \frac{1}{2}\Gamma_1]$

Therefore, $|\Gamma(\theta)| = |2||\Gamma_0 \cos(2\theta) + \frac{1}{2}\Gamma_1|$ with $\Gamma_0 = \Gamma_2$. Comparing with Eq.(3), we get $2\Gamma_0 = \frac{A}{2}$ and $\Gamma_1 = 0.6A$

Now, $\Gamma(0) = \frac{2\Gamma_0 - 1}{2\Gamma_0 + 1} = A(0.1 + 1)$ Therefore, $A = \frac{2}{3}$.

Again we know from the recursive formula that, $ln\frac{Z_{n+1}}{Z_n} = 2\Gamma_n$. So, by using this we get the characteristic impedance of the two sections of the transmission line as $Z_1 = 1.095Z_0$ and $Z_2 = 1.362Z_0$. Also, the two sections should have equal lengths.