Assignment 3

Week 3

Due on 04/04/16

Week 1

Week 2

Week 3

Question 1

The class of linear homogeneous differential equations of the form:

\[ \frac{dy}{dx} + P(x) y = Q(x) \]

is known as a first-order linear differential equation. The solution to this equation can be found by integrating the following expression:

\[ \int Q(x) e^{\int P(x) dx} dx \]

Question 2

Given the following system of differential equations:

\[ \begin{align*}
\frac{dx}{dt} &= -2x + 3y \\
\frac{dy}{dt} &= 2x - y
\end{align*} \]

Find the general solution for the given system.

Question 3

Consider the following differential equation:

\[ \frac{d^2y}{dx^2} + 4x \frac{dy}{dx} + 4y = 0 \]

Determine the characteristic equation and solve for the general solution.

Question 4

Given the initial value problem:

\[ \begin{align*}
\frac{dy}{dx} &= y \\
y(0) &= 1
\end{align*} \]

Find the solution to the differential equation.

Question 5

Consider the Laplace transform of the function:

\[ f(t) = e^{-2t} \sin(3t) \]

Determine the Laplace transform \( F(s) \) and then find the inverse Laplace transform to obtain \( f(t) \).

Question 6

The heat equation is a partial differential equation of the form:

\[ \frac{\partial u}{\partial t} = \alpha \frac{\partial^2 u}{\partial x^2} \]

where \( u(x,t) \) is the temperature distribution, \( t \) is time, \( x \) is position, and \( \alpha \) is the diffusivity. State the initial and boundary conditions for the heat equation.

Question 7

Consider the following system of linear equations:

\[ \begin{align*}
2x + 3y &= 7 \\
4x - y &= 5
\end{align*} \]

Solve for the values of \( x \) and \( y \).

Question 8

Given the system:

\[ \begin{align*}
\frac{dx}{dt} &= 2x - y \\
\frac{dy}{dt} &= x + 4y
\end{align*} \]

Find the eigenvalues and eigenvectors of the matrix associated with this system. Use these to find the general solution.

Question 9

Consider the function:

\[ f(x) = \frac{1}{x} \]

Determine the domain of \( f(x) \) and find the derivative of \( f(x) \).

Question 10

For the wave function \( \psi(x,t) \) given by:

\[ \psi(x,t) = A e^{i(kx - \omega t)} \]

Find the spatial and temporal frequencies as well as the wave number.

Question 11

The gravitational potential energy \( U \) of two masses \( m_1 \) and \( m_2 \) separated by a distance \( r \) is given by:

\[ U = -\frac{G m_1 m_2}{r} \]

Compute the change in potential energy if the distance between the masses is doubled.

Question 12

Consider the following function:

\[ f(x) = \begin{cases} 
1 & \text{if } x < 0 \\
0 & \text{if } x = 0 \\
-1 & \text{if } x > 0
\end{cases} \]

Determine the derivative of \( f(x) \) at \( x = 0 \).