1. Consider MMSE estimation for the wireless sensor network (WSN) scenario as described in lectures with each observation \( y(k) = h + v(k) \), for \( 1 \leq k \leq N \), i.e. number of observations \( N \) and IID Gaussian noise samples of variance \( \sigma^2 \). What is the expression for the MMSE estimate \( \hat{h} \) of the unknown parameter \( h \)

   a. \( \hat{h} = \frac{(1^T y)/N \cdot \mu_h}{\sigma^2/N + \frac{1}{\sigma_h^2}} \)

   b. \( \hat{h} = \frac{(1^T y)/N \cdot \mu_h}{\sigma^2/N + \sigma_h^2} \)

   c. \( \hat{h} = \frac{(1^T y)/N \cdot \mu_h}{\sigma^2/N + \sigma_h^2} \)

   d. \( \hat{h} = \frac{(1^T y)/N \cdot \mu_h}{\sigma^2 + \sigma_h^2} \)

   Ans b

2. Consider MMSE estimation for the wireless sensor network (WSN) scenario as described in lectures with each observation \( y(k) = h + v(k) \), for \( 1 \leq k \leq N \), i.e. number of observations \( N \) and IID Gaussian noise samples of dB variance \( \sigma^2 \). As the number of observations \( N \to \infty \), the MMSE estimate \( \hat{h} \) of the unknown parameter \( h \) tends to,

   a. 0

   b. 1

   c. ML Estimate

   d. Prior Mean

   Ans c

3. Consider MMSE estimation for the wireless sensor network (WSN) scenario as described in lectures with each observation \( y(k) = h + v(k) \), for \( 1 \leq k \leq 5 \), i.e. number of observations \( N = 5 \) and IID Gaussian noise samples of dB variance \( \sigma^2 = -3 \) dB i.e. \( 10\log_{10} \sigma^2 = -3 \). Let the observations be \( y(1) = 1, y(2) = 1, y(3) = 2, y(4) = 3/2, y(5) = 5/2 \).

   Let \( \mu_h = 1/2 \) and \( \sigma_h^2 = 1/4 \). What is the maximum likelihood estimate \( \hat{h} \) of the unknown parameter \( h \)

   a. 1.6

   b. 1

   c. 1.5

   d. 2
4. Consider MMSE estimation for the wireless sensor network (WSN) scenario as described in lectures with each observation \( y(k) = h + v(k) \), for \( 1 \leq k \leq 5 \), i.e. number of observations \( N = 5 \) and IID Gaussian noise samples of dB variance \( \sigma^2 = -3 \) dB i.e. \( 10\log_{10} \sigma^2 = -3 \) dB. Let the observations be \( y(1) = 1, y(2) = 1, y(3) = 2, y(4) = 3/2, y(5) = 5/2 \). Let \( \mu_h = 1/2 \) and \( \sigma_h^2 = 1/4 \). What is the MMSE estimate \( \hat{h} \) of the unknown parameter \( h \)
   a. \( 10/7 \)
   b. \( 8/7 \)
   c. \( 5/7 \)
   d. \( 9/7 \)
   Ans d

5. Consider MMSE estimation for the wireless sensor network (WSN) scenario as described in lectures with each observation \( y(k) = h + v(k) \), for \( 1 \leq k \leq 5 \), i.e. number of observations \( N = 5 \) and IID Gaussian noise samples of dB variance \( \sigma^2 = -3 \) dB i.e. \( 10\log_{10} \sigma^2 = -3 \) dB. Let the observations be \( y(1) = 1, y(2) = 1, y(3) = 2, y(4) = 3/2, y(5) = 5/2 \). Let \( \mu_h = 1/2 \) and \( \sigma_h^2 = 1/4 \). What is the posterior probability density function of the unknown parameter \( h \)
   a. Gaussian
   b. Exponential
   c. Rayleigh
   d. Uniform
   Ans a

6. Given the fading channel estimation problem where the output symbol \( y(k) \) is \( y(k) = hx(k) + v(k) \), with \( h, x(k), v(k) \) denoting the real channel coefficient, pilot symbol and noise sample respectively. Let \( x = [x(1), x(2), \ldots, x(N)]^T \) denote the pilot vector of transmitted pilot symbols and \( y = [y(1), y(2), \ldots, y(N)]^T \) denote the corresponding received symbol vector. Let \( v(k) \) be IID Gaussian noise with zero-mean and variance \( \sigma^2 \). Let \( \mu_h, \sigma_h^2 \) denote the prior mean, variance of the parameter \( h \). The maximum likelihood estimate of the channel coefficient \( h \) is,
   a. \( \hat{h} = \frac{x^Ty}{\|x\|} \)
   b. \( \hat{h} = \frac{y^Ty}{\|x\|^2} \)
   c. \( \hat{h} = \frac{x^Ty}{\|x\|^2} \)
   d. \( \hat{h} = \frac{y^Ty}{\|x\|} \)
   Ans a
7. Given the fading channel estimation problem where the output symbol $y(k)$ is $y(k) = hx(k) + v(k)$, with $h$, $x(k)$, $v(k)$ denoting the real channel coefficient, pilot symbol and noise sample respectively. Let $\mathbf{x} = [x(1), x(2), \ldots, x(N)]^T$ denote the pilot vector of transmitted pilot symbols and $\mathbf{y} = [y(1), y(2), \ldots, y(N)]^T$ denote the corresponding received symbol vector. Let $v(k)$ be IID Gaussian noise with zero-mean and variance $\sigma^2$. Let $\mu_h$, $\sigma_h^2$ denote the prior mean, variance of the parameter $h$. The covariance matrix $R_{yy}$ of the output vector $\mathbf{y}$ is,

- $R_{yy} = \sigma^2 \mathbf{xx}^T + \sigma_h^2 \mathbf{I}$
- $R_{yy} = \mu_h \mathbf{xx}^T + \sigma^2 \mathbf{I}$
- $R_{yy} = \mathbf{xx}^T + \sigma^2 \mathbf{I}$
- $R_{yy} = \sigma_h^2 \mathbf{xx}^T + \mathbf{I}$

Ans b

8. Given the fading channel estimation problem where the output symbol $y(k)$ is $y(k) = hx(k) + v(k)$, with $h$, $x(k)$, $v(k)$ denoting the real channel coefficient, pilot symbol and noise sample respectively. Let $\mathbf{x} = [x(1), x(2), \ldots, x(N)]^T$ denote the pilot vector of transmitted pilot symbols and $\mathbf{y} = [y(1), y(2), \ldots, y(N)]^T$ denote the corresponding received symbol vector. Let $v(k)$ be IID Gaussian noise with zero-mean and variance $\sigma^2$. Let $\mu_h$, $\sigma_h^2$ denote the prior mean, variance of the parameter $h$. The MMSE estimate of the channel coefficient $h$ is,

- $\hat{h} = \frac{(\mathbf{x}^T \mathbf{y})/\|\mathbf{x}\|^2 \mu_h}{\sigma^2/\|\mathbf{x}\|^2 + \sigma_h^2}$
- $\hat{h} = \frac{(\mathbf{x}^T \mathbf{y})/\|\mathbf{x}\|^2 \mu_h}{\sigma^2/\|\mathbf{x}\|^2 + \sigma_h^2}$
- $\hat{h} = \frac{(\mathbf{x}^T \mathbf{y})/\|\mathbf{x}\|^2 \mu_h}{\sigma^2/\|\mathbf{x}\|^2 + \sigma_h^2}$
- $\hat{h} = \frac{(\mathbf{x}^T \mathbf{y})/\|\mathbf{x}\|^2 \mu_h}{\sigma^2/\|\mathbf{x}\|^2 + \sigma_h^2}$

Ans d

9. Consider the fading channel estimation problem where the output symbol $y(k)$ is $y(k) = hx(k) + v(k)$, with $h$, $x(k)$, $v(k)$ denoting the real channel coefficient, pilot symbol and noise sample respectively. Let $\mathbf{x} = \begin{bmatrix} 1/2 & 2 & 1 & 3/2 \end{bmatrix}^T$ denote the pilot vector of transmitted
pilot symbols and $y = [-2 \ 2 \ -1 \ 1]^T$ denote the corresponding received symbol vector. Let $v(k)$ be IID Gaussian noise with zero-mean and dB variance $\sigma^2 = 3$ dB. Let $\mu_h = 1$, $\sigma_h^2 = 1/2$ denote the prior mean, variance of the parameter $h$. The MMSE estimate of the channel coefficient $h$ is,

a. $15/23$

b. $20/23$

c. $21/23$

d. $17/23$

Ans a

10. Consider the fading channel estimation problem where the output symbol $y(k)$ is $y(k) = hx(k) + v(k)$, with $h$, $x(k)$, $v(k)$ denoting the real channel coefficient, pilot symbol and noise sample respectively. Let $x = \begin{bmatrix} 1/2 & 2 & 1 & 3/2 \end{bmatrix}^T$ denote the pilot vector of transmitted pilot symbols and $y = [-2 \ 2 \ -1 \ 1]^T$ denote the corresponding received symbol vector. Let $v(k)$ be IID Gaussian noise with zero-mean and variance $\sigma^2$. Let $\mu_h = 1$, $\sigma_h^2 = 1/2$ denote the prior mean, variance of the parameter $h$. As the noise variance $\sigma^2 \to \infty$, the MMSE estimate of the channel coefficient $h$ becomes,

a. $15/23$

b. $7/4$

c. $1$

d. $0$

Ans c