Assignment - 5

The due date for submitting this assignment has passed. Due on 2017-08-31, 23:59 IST. As per our records you have not submitted this assignment.

1) Consider the maximum likelihood (ML) multi-antenna channel estimation problem with \( N \) transmitted pilot vectors \( \mathbf{x}(k) \), pilot matrix \( \mathbf{X} \) and receive vector \( \mathbf{y} \). Let the channel vector be \( \mathbf{h} = [h_1, h_2, \ldots, h_M]^T \). Let the noise samples \( \mathbf{v}(k) \) be independent Gaussian with zero-mean and variance \( \sigma_k^2 \). Let \( \mathbf{R} \) denote the covariance matrix of the noise vector \( \mathbf{v} = [v(1), v(2), \ldots, v(N)]^T \). The ML estimate of the channel vector \( \mathbf{h} \) is,

\[
\begin{aligned}
\left( \mathbf{X} \mathbf{R}^{-1} \mathbf{X}^T \right)^{-1} & \mathbf{X}^T \mathbf{R}^{-1} \mathbf{y} \\
\left( \sum_{k=1}^{N} \frac{1}{\sigma_k^2} \mathbf{x}(k) \mathbf{x}(k)^T \right)^{-1} & \left( \sum_{k=1}^{N} \frac{1}{\sigma_k^2} \mathbf{x}(k) \mathbf{y}(k) \right) \\
\left( \sum_{k=1}^{N} \sigma_k^2 \mathbf{x}(k) \mathbf{x}(k)^T \right)^{-1} & \left( \sum_{k=1}^{N} \sigma_k^2 \mathbf{x}(k) \mathbf{y}(k) \right) \\
\mathbf{XR}^{-1} \mathbf{X}^T & \mathbf{R}^{-1} \mathbf{y} \\
\text{No, the answer is incorrect.} \\
\text{Score: 0}
\end{aligned}
\]

Accepted Answers:

\[
\left( \sum_{k=1}^{N} \frac{1}{\sigma_k^2} \mathbf{x}(k) \mathbf{x}(k)^T \right)^{-1} \left( \sum_{k=1}^{N} \frac{1}{\sigma_k^2} \mathbf{x}(k) \mathbf{y}(k) \right)
\]

2) Consider the MIMO channel estimation problem with pilot vectors \( \mathbf{x}(1) = [3, -2]^T, \mathbf{x}(2) = [-2, 3]^T, \mathbf{x}(3) = [4, 2]^T, \mathbf{x}(4) = [2, 2]^T \). The received output vectors \( \mathbf{y} \) are

\[
\begin{aligned}
\mathbf{y}(1) & = [-2, 1, -3]^T, \mathbf{y}(2) = [-1, 3, 3]^T, \mathbf{y}(3) = [-1, -2, 2]^T, \mathbf{y}(4) = [-3, -1, 1]^T.
\end{aligned}
\]

The size of the MIMO system is,

\[
\begin{array}{c}
3 \times 2 \\
2 \times 2 \\
2 \times 3
\end{array}
\]
Estimation for Wireless Communications – MIMO/OFDM Cellular and Sensor Networks - - Unit 6 - Week 5 - Inter Sy

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Interference ISI

As described in the lectures, the output matrix $\mathbf{y}$ output vectors $\mathbf{y}$ are $\mathbf{x}$. Consider the MIMO channel estimation problem with pilot vectors $\mathbf{x}(1) = [3, -2]^T, \mathbf{x}(2) = [-2, 3]^T, \mathbf{x}(3) = [4, 2]^T, \mathbf{x}(4) = [2, 2]^T$. The received output vectors $\mathbf{y}$ are $\mathbf{y}(1) = [-2, 1, -3]^T, \mathbf{y}(2) = [-1, 3, 3]^T, \mathbf{y}(3) = [-1, -2, 2]^T, \mathbf{y}(4) = [-3, -1, 1]^T$.

As described in the lectures, the pilot matrix $\mathbf{X}$ for the MIMO channel estimation problem above is,

$\mathbf{X} = \begin{bmatrix} 3 & -2 \\ -2 & 3 \\ 4 & 2 \\ 2 & 2 \end{bmatrix}$

$\mathbf{X} = \begin{bmatrix} 3 & -2 & 2 & 2 \\ 3 & 4 & 2 & 2 \end{bmatrix}$

$\mathbf{X} = \begin{bmatrix} 3 & -2 \\ -2 & 3 \end{bmatrix}$

No, the answer is incorrect.
Score: 0

Accepted Answers:
$\mathbf{X} = \begin{bmatrix} 3 & -2 \\ -2 & 3 \end{bmatrix}$

No, the answer is incorrect.
Score: 0

Accepted Answers:

$\mathbf{X} = \begin{bmatrix} 3 & -2 & 4 & 2 \\ -2 & 3 & 2 & 2 \end{bmatrix}$

4) Consider the MIMO channel estimation problem with pilot vectors $\mathbf{x}(1) = [3, -2]^T, \mathbf{x}(2) = [-2, 3]^T, \mathbf{x}(3) = [4, 2]^T, \mathbf{x}(4) = [2, 2]^T$. The received output vectors $\mathbf{y}$ are $\mathbf{y}(1) = [-2, 1, -3]^T, \mathbf{y}(2) = [-1, 3, 3]^T, \mathbf{y}(3) = [-1, -2, 2]^T, \mathbf{y}(4) = [-3, -1, 1]^T$.

As described in the lectures, the output matrix $\mathbf{Y}$ for the MIMO channel estimation problem above is,

$\mathbf{Y} = \begin{bmatrix} -2 & 1 & -3 & -1 \\ -1 & -2 & 2 & -3 \\ -1 & 3 & 3 \\ -3 & -1 & 1 \end{bmatrix}$

$\mathbf{Y} = \begin{bmatrix} -2 & 1 \\ -1 & 3 \\ -3 & -1 \end{bmatrix}$

$\mathbf{Y} = \begin{bmatrix} -2 & 1 & -3 \\ -1 & 3 & 3 \\ -3 & -1 \end{bmatrix}$

$\mathbf{Y} = \begin{bmatrix} -2 & 1 & -3 & -1 & 3 & 3 \\ -1 & -2 & 2 & -3 & -1 & 1 \end{bmatrix}$

https://onlinecourses-archive.nptel.ac.in/noc17_ee19/unit?unit=117&assessment=152
Consider the MIMO channel estimation problem with pilot vectors \( \mathbf{x}(1) = [3, -2]^T, \mathbf{x}(2) = [-2, 3]^T, \mathbf{x}(3) = [4, 2]^T, \mathbf{x}(4) = [2, 2]^T \). The received output vectors \( \mathbf{y} \) are \( \mathbf{y}(1) = [-2, 1, -3]^T, \mathbf{y}(2) = [-1, 3, 3]^T, \mathbf{y}(3) = [-1, -2, 2]^T, \mathbf{y}(4) = [-3, -1, 1]^T \).

The LS estimate of the MIMO channel matrix is given as,

\[
\mathbf{Y} = \begin{bmatrix}
-2 & 1 & -3 \\
-1 & 3 & 3 \\
-1 & -2 & 2 \\
-3 & -1 & 1
\end{bmatrix}
\]

1) No, the answer is incorrect.

Score: 0

Accepted Answers:

\[ \mathbf{Y} \mathbf{x} (\mathbf{x}^T \mathbf{x})^{-1} \]
\[ (\mathbf{x}^T \mathbf{x})^{-1} \mathbf{x}^T \mathbf{y} \]
\[ \mathbf{y} \mathbf{x} (\mathbf{x}^T \mathbf{x})^{-1} \]
\[ (\mathbf{x}^T \mathbf{x})^{-1} \mathbf{x}^T \mathbf{y} \]

No, the answer is incorrect.

Score: 0

Accepted Answers:

\[ \mathbf{Y} \mathbf{x} (\mathbf{x}^T \mathbf{x})^{-1} \]

6) No, the answer is incorrect.

Score: 0

Accepted Answers:
7) Consider the MIMO channel estimation problem with pilot vectors $\mathbf{x}(1) = [3, -2]^T$, $\mathbf{x}(2) = [-2, 3]^T$, $\mathbf{x}(3) = [4, 2]^T$, $\mathbf{x}(4) = [2, 2]^T$. The received output vectors $\mathbf{y}$ are $\mathbf{y}(1) = [-2, 1, -3]^T$, $\mathbf{y}(2) = [1, 3, 3]^T$, $\mathbf{y}(3) = [-1, -2, 2]^T$, $\mathbf{y}(4) = [-3, -1]^T$. The estimate of the MIMO channel matrix $\mathbf{H}$ is

$$\frac{1}{33} \begin{bmatrix} 21 & -17 & 8 \\ 13 & 20 & 17 \end{bmatrix}$$

No, the answer is incorrect.
Score: 0

Accepted Answers:
$$\begin{bmatrix} \frac{14}{33} & \frac{7}{21} \\ \frac{13}{33} & \frac{1}{21} \\ \frac{5}{33} & \frac{1}{21} \end{bmatrix}$$

8) Consider the MIMO channel estimation problem with pilot vectors $\mathbf{x}(1) = [3, -2]^T$, $\mathbf{x}(2) = [-2, 3]^T$, $\mathbf{x}(3) = [4, 2]^T$, $\mathbf{x}(4) = [2, 2]^T$. The received output vectors $\mathbf{y}$ are $\mathbf{y}(1) = [-2, 1, -3]^T$, $\mathbf{y}(2) = [1, 3, 3]^T$, $\mathbf{y}(3) = [-1, -2, 2]^T$, $\mathbf{y}(4) = [-3, -1, 1]^T$. Let the noise samples be IID Gaussian zero-mean with variance $-6dB$. What are the variances of the estimates of coefficients in any row of the MIMO channel matrix?

$$\begin{bmatrix} \frac{1}{66} & \frac{1}{98} \\ \frac{1}{132} & \frac{1}{84} \\ \frac{1}{4} \\ \frac{5}{36} & \frac{9}{88} \end{bmatrix}$$

No, the answer is incorrect.
Score: 0

Accepted Answers:
$$\begin{bmatrix} \frac{1}{132} & \frac{1}{84} \end{bmatrix}$$

9) Channel equalization refers to
Consider an Inter Symbol Interference channel $y(k) = x(k) + \frac{1}{3}x(k - 1) + v(k)$. Let an $r = 2$ tap channel equalizer be designed for this scenario based on symbols $y(k), y(k + 1)$ to detect $x(k)$. What is the effective channel matrix $H$ for this scenario?

- $\begin{bmatrix} 1 & \frac{1}{3} & 0 \\ 0 & 1 & \frac{1}{3} \end{bmatrix}$
- $\begin{bmatrix} 1 & \frac{1}{3} \\ \frac{1}{3} & 1 \end{bmatrix}$
- $\begin{bmatrix} \frac{1}{3} & 1 & 0 \\ 0 & \frac{1}{3} & 1 \end{bmatrix}$
- $\begin{bmatrix} 1 & \frac{1}{3} \\ \frac{1}{3} & 1 \end{bmatrix}$

No, the answer is incorrect.
Score: 0
Accepted Answers:
- $\begin{bmatrix} 1 & \frac{1}{3} & 0 \\ 0 & 1 & \frac{1}{3} \end{bmatrix}$