Assignments 1

The due date for submitting this assignment has passed. Due on 2017-02-08, 23:30 IST. As per our records you have not submitted this assignment.

1) Question 1: We know that any signal $x(t) \in L_2(\mathbb{R})$ (i.e. signal $x(t)$ having finite energy) can be expanded

$$x(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} \hat{X}(\Omega) e^{j\Omega t} d\Omega,$$

where $\hat{X}(\Omega)$ is the Fourier transform of signal $x(t)$. Then which of the following is true about time-frequency localization of the Fourier basis element?

- The Fourier basis element has good time localization but poor frequency localization.
- The Fourier basis element has poor time localization but good frequency localization.
- Since we can expand any finite energy signal $x(t)$ into Fourier basis, it must have basis element exhibiting good time and frequency localizations.
- None of the above.

No, the answer is incorrect.

Score: 0

Accepted Answers:
The Fourier basis element has poor time localization but good frequency localization.

2) Question 2: Which of the following forms the basis vectors for the 3 dimensional vector space $\mathbb{R}^3$?

$$\begin{bmatrix} 1 & 2 & 6 \\ 2 & 4 & 12 \\ 3 & 6 & 18 \end{bmatrix}$$

- $egin{bmatrix} 4 & 6 & 14 \\ 6 & 12 & 24 \\ 11 & 21 & 43 \end{bmatrix}$

- $egin{bmatrix} 0 & 0 & 0 \\ 2 & 2 & 2 \\ 4 & 1 & 0 \end{bmatrix}$

- $egin{bmatrix} 0 & 4 & 2 \\ 2 & 0 & 2 \\ 2 & 4 & 0 \end{bmatrix}$
3) Question 3:
Which of the following functions belong to $L_2(\mathbb{R})$? (Multiple options can be correct)

- $f(t) = \begin{cases} \frac{1}{t} & \text{if } t \geq 1 \\ 0 & \text{otherwise} \end{cases}$
- $f(t) = \begin{cases} \frac{1}{t^2} & \text{if } t \geq 1 \\ 0 & \text{otherwise} \end{cases}$
- $f(t) = \begin{cases} \frac{1}{\sqrt{t}} & \text{if } t > 0 \\ 0 & \text{otherwise} \end{cases}$
- $f(t) = \begin{cases} t & \text{if } t \geq 0 \\ 0 & \text{otherwise} \end{cases}$

No, the answer is incorrect.
Score: 0

Accepted Answers:

4) Question 4: Suppose that we don’t use Fourier basis as our basis elements and instead decide to use some other basis elements composed of shifted Dirac deltas $\delta(t - k)$, where $k \in \mathbb{Z}$. Then which of the following is true about time-frequency localization of this new basis element?

- It has good time localization but poor frequency localization.
- It has poor time localization but good frequency localization.
- It exhibits good time as well as frequency localizations.
- None of the above.

No, the answer is incorrect.
Score: 0

Accepted Answers:

5) Question 5: Given $x[n] = 3^{-n}u[n]$. Find $l_2$ norm of the sequence.

No, the answer is incorrect.
Score: 0

Accepted Answers:

It has good time localization but poor frequency localization.
6) Question 6: Suppose we are given two functions \( x_1(t) \) and \( x_2(t) \) as
\[
 x_1(t) = \cos(10\pi t) \quad \text{and} \quad x_2(t) = e^{-t^2} \cos(10t)
\]
Then which of the signals represents a wavelet or a "small wave" as discussed in the lecture?

- \( x_2(t) \)
- \( x_1(t) \)
- \( x_1(t) \) and \( x_2(t) \)
- None of these

No, the answer is incorrect.
Score: 0
Accepted Answers:
\[
\frac{3}{2\sqrt{2}}
\]

7) Question 7: What is the Fourier transform of the following function?
\[
f(t) = \begin{cases} e^{-|a|} & \text{if } |t| < 1, \quad a > 0 \\ 0 & \text{otherwise} \end{cases}
\]
\[
\begin{align*}
\frac{2a}{a^2 + \Omega^2} - \frac{e^{-(a+j\Omega)}}{a + j\Omega} - & \frac{e^{-(a-j\Omega)}}{a - j\Omega} \\
\frac{2j\Omega}{a^2 + \Omega^2} + \frac{e^{-(a+j\Omega)}}{a + j\Omega} - & \frac{e^{-(a-j\Omega)}}{a - j\Omega} \\
\frac{2j\Omega}{a + \Omega^2} + & \frac{e^{-(a+j\Omega)}}{a + j\Omega} - \frac{e^{-(a-j\Omega)}}{a - j\Omega} \\
\frac{2a}{a^2 + \Omega^2} - & \frac{e^{-(a+j\Omega)}}{a + j\Omega} - \frac{e^{-(a-j\Omega)}}{a - j\Omega}
\end{align*}
\]

No, the answer is incorrect.
Score: 0
Accepted Answers:
\[
\frac{2a}{a^2 + \Omega^2} - \frac{e^{-(a+j\Omega)}}{a + j\Omega} - \frac{e^{-(a-j\Omega)}}{a - j\Omega}
\]

8) Question 8: Which of the following functions have finite energy?

No, the answer is incorrect.
Score: 0
Accepted Answers:
\[
\frac{2a}{a^2 + \Omega^2} - \frac{e^{-(a+j\Omega)}}{a + j\Omega} - \frac{e^{-(a-j\Omega)}}{a - j\Omega}
\]
9) Question 9: Find the $L_4$-norm of the following function:

\[ f(x) = \begin{cases} 
    x^2 & \text{if } |x| < 1 \\
    0 & \text{otherwise}
  \end{cases} \]

\[ L_4(x) = \begin{cases} 
    x^2 & \text{if } |x| < 1 \\
    0 & \text{otherwise}
  \end{cases} \]

No, the answer is incorrect.
Score: 0
Accepted Answers:
\[ \left( \frac{2}{5} \right)^{1/4} \]
\[ \left( \frac{1}{5} \right)^{1/4} \]
\[ \left( \frac{2}{9} \right)^{1/4} \]
\[ \left( \frac{1}{2} \right)^{1/4} \]

No, the answer is incorrect.
Score: 0
Accepted Answers:
\[ \left( \frac{2}{9} \right)^{1/4} \]

10) Question 10: Let $\phi(t)$ and $\psi(t)$ be the Haar scaling and wavelet functions. Which one of the following relationship holds?

\[ \psi(t) = \phi(2t) - \phi(2t - 1) \]
\[ \psi(t) = \phi(t) - \phi(t - 1) \]
\[ \psi(t) = \phi \left( \frac{t}{2} \right) - \phi \left( \frac{t}{2} + 1 \right) \]
\[ \psi(t) = \phi(t) + \phi(2t - 1) \]

No, the answer is incorrect.
Score: 0
Accepted Answers:
\[ \psi(t) = \phi(2t) - \phi(2t - 1) \]

11) Question 11: Which of the following are the standard basis functions for the Fourier Transform?

\[ f(t) = \begin{cases} 
    1 & \text{if } t \geq 0 \\
    -1 & \text{otherwise}
  \end{cases} \]

\[ f(t) = \sin(t) \]

\[ f(t) = \begin{cases} 
    t^2 & \text{if } t \geq 0 \\
    0 & \text{otherwise}
  \end{cases} \]

\[ f(t) = \begin{cases} 
    e^{-t} & \text{if } t \geq 0 \\
    0 & \text{otherwise}
  \end{cases} \]
Complex exponential functions $e^{j\Omega t}$, $\Omega \in \mathbb{R}$.

The set of sinusoidal functions $\{\sin(\Omega t), \cos(\Omega t)\}$, $\Omega \in \mathbb{R}$.

All the above.

None of the above.

**No, the answer is incorrect.**

**Score:** 0

**Accepted Answers:**

All the above.

12 Question 12: Let $x[n] = e^{-2n}u[n]$ be the input to a system. Which of the following impulse responses gives the bounded output for this input?

- $h[n] = n$
- $h[n] = 2^n u[n]$
- $h[n] = \frac{1}{n}$
- $h[n] = \frac{1}{n^2}$

**No, the answer is incorrect.**

**Score:** 0

**Accepted Answers:**

$h[n] = \frac{1}{n^2}$