Assignment 12

The due date for submitting this assignment has passed.

Due on 2021-04-14, 23:59 BST.

1. The following statement concerns an ideal Bose gas in a harmonic trap. Which of these false? (Mark 100 points for Bose-Einstein Condensation)

(a) In two dimensions, BEC is only possible at zero temperature.
(b) In two dimensions, there is no BEC in the thermodynamic limit.
(c) In one dimension, BEC is only possible at zero temperature.
(d) In one dimension, there is no BEC in the thermodynamic limit.

No, the answer is incorrect.

Bose-Einstein condensation occurs in one dimension, with no temperature limit.

2. A gas of $N = 10^5$ atoms is placed in an asymmetric harmonic trap with conical frequencies $\omega_x = 205$, $\omega_y = 409$ and $\omega_z = 793$. The conformation temperature $T$, is around

\[ T \approx 10^8 \text{ K} \]

\[ T \approx 10^7 \text{ K} \]

\[ T \approx 10^5 \text{ K} \]

\[ 10^{-5} \text{ K} \]

No, the answer is incorrect.

Bose-Einstein condensation temperature is $T_c = \frac{\hbar}{k_B} \sqrt{\frac{\omega_z}{2}}$, where $\hbar$ is the reduced Planck constant.

3. If we experimentally measure $\rho(T)$ as a function of transition density $\rho$, we obtain a straight line with a slope of $\frac{1}{3}$ in the log-log plot i.e.,

\[ \rho(T) = \rho_0 (T) \frac{1}{3} \]

\[ \rho(T) = \rho_0 (T) \frac{1}{5} \]

\[ \rho(T) = \rho_0 (T) \frac{1}{7} \]

\[ \rho(T) = \rho_0 (T) \frac{1}{100} \]

No, the answer is incorrect.

Bose-Einstein condensation occurs when $\rho(T) \propto T^{-\frac{1}{3}}$.

4. For the case of $T > T_c$, the expression of pressure $p(T)$, where $\rho = \rho_0$ is the liquid is

\[ p(T,\rho_0) = \frac{\hbar^2}{2\pi^2 m} \int_{-\infty}^{\infty} \frac{dx}{\omega_x^2} \int_{-\infty}^{\infty} \frac{dy}{\omega_y^2} \int_{-\infty}^{\infty} \frac{dz}{\omega_z^2} \sum_{n_x, n_y, n_z} \frac{1}{n_x n_y n_z} \omega_x \omega_y \omega_z \]

\[ p(T,\rho_0) = \frac{\hbar^2}{2\pi^2 m} \int_{-\infty}^{\infty} \frac{dx}{\omega_x^2} \int_{-\infty}^{\infty} \frac{dy}{\omega_y^2} \int_{-\infty}^{\infty} \frac{dz}{\omega_z^2} \sum_{n_x, n_y, n_z} \frac{1}{n_x n_y n_z} \omega_x \omega_y \omega_z \]

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\[ p(T,\rho_0) = \frac{\hbar^2}{2\pi^2 m} \int_{-\infty}^{\infty} \frac{dx}{\omega_x^2} \int_{-\infty}^{\infty} \frac{dy}{\omega_y^2} \int_{-\infty}^{\infty} \frac{dz}{\omega_z^2} \sum_{n_x, n_y, n_z} \frac{1}{n_x n_y n_z} \omega_x \omega_y \omega_z \]

No, the answer is incorrect.

The expression for pressure in the Bose-Einstein condensation is $p(T,\rho_0) \propto T^{-\frac{1}{3}}$.

5. For $T < T_c$, the value of the heat capacity is

\[ C_v = \frac{\hbar^2}{2\pi^2 m} \int_{-\infty}^{\infty} \frac{dx}{\omega_x^2} \int_{-\infty}^{\infty} \frac{dy}{\omega_y^2} \int_{-\infty}^{\infty} \frac{dz}{\omega_z^2} \sum_{n_x, n_y, n_z} \frac{1}{n_x n_y n_z} \omega_x \omega_y \omega_z \]

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No, the answer is incorrect.

Bose-Einstein condensation heat capacity is $C_v \propto T^{-\frac{2}{3}}$.

6. The single-particle density of states $g(\epsilon)$ is

\[ g(\epsilon) = \frac{\epsilon}{\hbar^2} \frac{\sqrt{\epsilon^2 - \epsilon^2}}{\epsilon^2} \frac{\sqrt{\epsilon^2 - \epsilon^2}}{\epsilon^2} \]

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No, the answer is incorrect.

Bose-Einstein condensation single-particle density of states $g(\epsilon) = \frac{\epsilon}{\hbar^2} \frac{\sqrt{\epsilon^2 - \epsilon^2}}{\epsilon^2} \frac{\sqrt{\epsilon^2 - \epsilon^2}}{\epsilon^2}$.

7. For photon statistics, the density of states $g(\epsilon)$ is $g(\epsilon) = \frac{\epsilon}{\hbar^2} \frac{\sqrt{\epsilon^2 - \epsilon^2}}{\epsilon^2} \frac{\sqrt{\epsilon^2 - \epsilon^2}}{\epsilon^2}$. The degeneracy factor is $g = 1$ and the number density is $n^2$.

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