Assignment 7

Due on 2020-03-18, 23:59 EST.

The deadline for submitting this assignment has passed.

As per our records, you have not submitted this assignment.

Method of images with spherical conductor

Using the law of images, the potential outside a grounded spherical conductor may be expressed in spherical coordinates as

\[
V(r, \theta, \phi) = \frac{Q}{4\pi\epsilon_0} \left( \frac{1}{r - a} - \frac{1}{r + a} \right)
\]

where \(a\) is the radius of the spherical conductor, and \(r + a\) is the distance of the point charge from the center of the sphere. \(V = 0\) for the sphere \(r = a\).

1. What is the induced surface charge density on the surface of the sphere? 5 points

\[
\sigma = \frac{1}{2 \pi a^2} \left( \frac{1}{r} \right) \frac{dV}{dr}
\]

2. What is the total amount of induced charge (positive \(\sigma\)) to find this? 1 points

\[
\sigma = \frac{Q}{4\pi a^2}
\]

3. How much work is done to bring the point charge, \(q\), from infinity to distance \(a\) from the center of the sphere? 5 points

\[
W = \int_{r = \infty}^{r = a} \sigma \, dV = \frac{1}{4\pi \epsilon_0} \int_{r = \infty}^{r = a} \frac{1}{r} \, dV
\]

4. Potential of an electric multipole

A sphere of radius \(R\), centered at the origin, carries a charge density

\[
\rho(r, \theta, \phi) = \frac{Q}{4\pi R^3} \left( \frac{1}{r} \right) \frac{dV}{dr}
\]

where \(Q\) is a constant, \(r\) are the usual spherical coordinates. What is the approximate potential for points on the \(z\)-axis far from the sphere? 5 points

\[
V(0, \theta, \phi) = \frac{1}{4\pi \epsilon_0} \int_{r = R}^{\infty} \frac{1}{r} \, dV = \frac{1}{4\pi \epsilon_0} \frac{Q}{R^2}
\]

5. First nonzero term in multipole expansion

A disc, ring of radius \(R\), centered on the origin on the xy plane carries a uniform linear charge density \(\lambda\). What is the first nonzero term in the multipole expansion for the potential \(V(0, \theta, \phi)\) at \(r = a\)? 7 points

\[
V_{1}(a, \theta, \phi) = \frac{1}{4\pi \epsilon_0} \int_{r = R}^{a} \frac{1}{r} \, dV = \frac{1}{4\pi \epsilon_0} \frac{\lambda a^2}{2}
\]