Unit 8 - Week 7

Course outline

Assignment 7

The due date for submitting this assignment has passed.
As per our records you have not submitted this assignment.

Due on 2020-03-18, 23:59 IST.

Method of images with spherical conductor

Using the law of cosines, the potential outside a grounded spherical conductor may be expressed in spherical coordinates as

$$V(r,\theta)=\frac{1}{4\pi\epsilon_0}\left[\frac{q}{\sqrt{r^2+a^2-2ra\cos\theta}}-\frac{q}{\sqrt{R^2+(ra/R)^2-2ra\cos\theta}}\right],$$

where r and θ are the usual spherical polar coordinates, with the z-axis along the line through the point charge q. R is the radius of the conducting sphere and a is the distance of the point charge from the center of the sphere. $V=0$ on the sphere $r=R$.

1) What is the induced surface charge density on the surface of the sphere? *5 points*

- $\sigma(\theta)=\frac{q}{4\pi R}(R^2-a^2)(R^2+a^2-2Ra\cos\theta)^{-3/2}$
- $\sigma(\theta)=\frac{q}{4\pi R}(R^2+a^2)(R^2+a^2-2Ra\cos\theta)^{-1/2}$
- $\sigma(\theta)=\frac{q}{4\pi a}(R^2-a^2)(R^2+a^2+2Ra\cos\theta)^{1/2}$
- $\sigma(\theta)=\frac{q}{4\pi R}(R^2+a^2)(R^2+a^2-2Ra\sin\theta)^{-1/2}$

No, the answer is incorrect.
Score: 0
Accepted Answers:
$\sigma(\theta)=\frac{q}{4\pi R}(R^2-a^2)(R^2+a^2-2Ra\cos\theta)^{-3/2}$

2) What is the total amount of induced charge (integrate $\sigma(\theta)$ to find this)? *5 points*

- qR/a
- qa/R
- $-qR/a$
- $-qa/R$

No, the answer is incorrect.
Score: 0
Accepted Answers:
$-qR/a$

3) How much work is done to bring the point charge q from infinity to distance a from the center of the sphere? *5 points*

- Work done = $\frac{1}{4\pi\epsilon_0}\frac{qR}{(a^2-R^2)}$
- Work done = $-\frac{1}{4\pi\epsilon_0}\frac{qR}{2(a^2+R^2)}$
- Work done = $\frac{1}{4\pi\epsilon_0}\frac{qR}{2(a^2-R^2)}$
- Work done = $-\frac{1}{4\pi\epsilon_0}\frac{q^2R}{2(a^2-R^2)}$

No, the answer is incorrect.
Score: 0
Accepted Answers:
Work done = $-\frac{1}{4\pi\epsilon_0}\frac{q^2R}{2(a^2-R^2)}$

4) **Potential of electric multipole** *8 points*

A sphere of radius R, centered at the origin, carries a charge density

$$\rho(r,\theta)=k\frac{R}{r^2}(R-2r)\sin\theta$$

where k is a constant, r, θ are the usual spherical coordinates. What is the approximate potential for points on the z-axis far from the sphere?

- $V(z)\cong\frac{1}{4\pi\epsilon_0}\frac{k\pi^2R^4}{24z^3}$
- $V(z)\cong\frac{1}{4\pi\epsilon_0}\frac{k\pi^2R^5}{48z^3}$
- $V(z)\cong-\frac{1}{4\pi\epsilon_0}\frac{k\pi^2R^4}{24z^3}$
- $V(z)\cong\frac{1}{4\pi\epsilon_0}\frac{k\pi^2z^4}{84R^3}$

No, the answer is incorrect.
Score: 0
Accepted Answers:
$V(z)\cong\frac{1}{4\pi\epsilon_0}\frac{k\pi^2R^5}{48z^3}$

5) **First nonzero term in multipole expansion** *7 points*

A circular ring of radius R, centered at the origin on the xy plane carries a uniform line charge density λ. What is the first nonzero term in the multipole expansion for the potential $V(r,\theta)$?

- $V^{first}_{nonzero}=-\frac{\lambda}{8\epsilon_0}\frac{R^3}{r^3}(3\cos^2\theta-1)$
- $V^{first}_{nonzero}=\frac{\lambda}{4\pi\epsilon_0}\frac{R^3}{r^3}(3\sin^2\theta-1)$
- $V^{first}_{nonzero}=-\frac{\lambda}{4\pi\epsilon_0}\frac{R^3}{r^2}(3\cos^2\theta-1)$
- $V^{first}_{nonzero}=-\frac{\lambda}{8\epsilon_0}\frac{R^3}{r^2}(3\sin^2\theta+1)$

No, the answer is incorrect.
Score: 0
Accepted Answers:
$V^{first}_{nonzero}=-\frac{\lambda}{8\epsilon_0}\frac{R^3}{r^3}(3\cos^2\theta-1)$