

# Unit 3 - Week 2

## Course outline

How does an NPTEL online course work?

Week 1

Week 2

- More problems on vector differential calculus
- Vector integral calculus: Line integral
- Surface integral
- Volume integral
- Fundamental theorems of vector calculus: The gradient theorem
- The divergence theorem (Gauss's theorem)
- The curl theorem (Stokes' theorem)
- Week 2 Practice Assignment
- Quiz : Assignment 2
- Week 2 Feedback
- Assignment 2 solutions

Week 3

Week 4

Week 5

Week 6

Week 7

Week 8

Week 9

Week 10

Week 11

Week 12

Download Videos

Lecture materials

## Assignment 2

The due date for submitting this assignment has passed. As per our records you have not submitted this assignment.

Due on 2020-02-12, 23:59 IST.

### Line integral

Suppose  $\phi = 2xyz^2$ ,  $\vec{F} = xy\hat{x} - z\hat{y} + x^2\hat{z}$ , and  $C$  is a curve  $x = t^2, y = 2t, z = t^3$  from  $t = 0$  to  $t = 1$

1) The integral  $\int_C \phi d\vec{r}$  evaluates to

3 points

- $\frac{8}{11}\hat{x} + \frac{4}{5}\hat{y} + \frac{1}{2}\hat{z}$
- $\frac{8}{11}\hat{x} + \frac{4}{5}\hat{y} + \hat{z}$
- $\frac{8}{11}\hat{x} - \frac{4}{5}\hat{y} + \hat{z}$
- $\frac{8}{11}\hat{x} - \frac{4}{5}\hat{y} + \frac{1}{2}\hat{z}$

No, the answer is incorrect. Score: 0

Accepted Answers:  $\frac{8}{11}\hat{x} + \frac{4}{5}\hat{y} + \hat{z}$

2) The integral  $\int_C \vec{F} \times d\vec{r}$  evaluates to

4 points

- $\frac{9}{10}\hat{x} - \frac{2}{3}\hat{y} + \frac{7}{5}\hat{z}$
- $-\frac{9}{10}\hat{x} - \frac{2}{3}\hat{y} - \frac{7}{5}\hat{z}$
- $-\frac{9}{10}\hat{x} - \frac{2}{3}\hat{y} + \frac{7}{5}\hat{z}$
- $-\frac{9}{10}\hat{x} + \frac{2}{3}\hat{y} + \frac{7}{5}\hat{z}$

No, the answer is incorrect. Score: 0

Accepted Answers:  $-\frac{9}{10}\hat{x} - \frac{2}{3}\hat{y} + \frac{7}{5}\hat{z}$

3) Surface integral

8 points

Suppose  $\vec{A} = 18z\hat{x} - 12y\hat{y} + 3y\hat{z}$  and  $S$  is the part of the plane  $2x + 3y + 6z = 12$  located in the first octant.  $\int_S \vec{A} \cdot d\vec{a}$  ( $d\vec{a}$  is the area element) becomes

- 24
- 19
- 14
- 16

No, the answer is incorrect. Score: 0

Accepted Answers: 24

4) Volume integral

6 points

A volume  $V$  is bounded by the surfaces  $x = 0, y = 0, y = 6, z = x^2, z = 4$ . The volume integral  $\int_V \vec{F} dV$  for  $\vec{F} = 2xz\hat{x} - x\hat{y} + y^2\hat{z}$  is

- $125\hat{x} - 24\hat{y} + 384\hat{z}$
- $128\hat{x} + 24\hat{y} + 384\hat{z}$
- $125\hat{x} + 24\hat{y} + 384\hat{z}$
- $128\hat{x} - 24\hat{y} + 384\hat{z}$

No, the answer is incorrect. Score: 0

Accepted Answers:  $128\hat{x} - 24\hat{y} + 384\hat{z}$

5) Divergence theorem

8 points

A fluid of density  $\rho(x, y, z, t)$  moves with velocity  $\vec{v}(x, y, z, t)$ . The current density  $\vec{J}$  is given by  $\vec{J} = \rho\vec{v}$ . If there are no sources or sinks, applying divergence theorem the relation you find between  $\vec{\nabla} \cdot \vec{J}$  and  $\frac{\partial \rho}{\partial t}$  is

- $\vec{\nabla} \cdot \vec{J} = \frac{\partial \rho}{\partial t}$
- $\vec{\nabla} \cdot \vec{J} = -\frac{\partial \rho}{\partial t}$
- $\vec{\nabla} \cdot \vec{J} = \frac{1}{2} \frac{\partial \rho}{\partial t}$
- $\vec{\nabla} \cdot \vec{J} = \frac{1}{2} \frac{\partial \rho}{\partial t} v$

No, the answer is incorrect. Score: 0

Accepted Answers:  $\vec{\nabla} \cdot \vec{J} = -\frac{\partial \rho}{\partial t}$

6) Stokes' theorem

5 points

Let  $x = \cos t, y = \sin t, z = 0, 0 \leq t < 2\pi$  be the parametric equation of a line  $C$ . For a  $\vec{A} = (2x - y)\hat{x} - yz^2\hat{y} - y^2z\hat{z}$ , the surface integral  $\int_S (\vec{\nabla} \times \vec{A}) \cdot d\vec{a}$  over any surface  $S$  enclosed by the line  $C$  becomes

- $2\pi$
- $-2\pi$
- $\pi$
- $4\pi$

No, the answer is incorrect. Score: 0

Accepted Answers:  $\pi$