Assignment 6

1. Figure out the following derivative expressions:

a. \( f(x) = \frac{x^2}{3} \)

\[ f'(x) = \frac{2x}{3} \]

b. \( g(x) = \frac{1}{x} \)

\[ g'(x) = -\frac{1}{x^2} \]

c. \( h(x) = x^3 + 2x^2 - 5x + 1 \)

\[ h'(x) = 3x^2 + 4x - 5 \]

2. What is the slope of the tangent line at the given point?

a. \( f(x) = x^2 + 3x - 4 \) at \( x = 2 \)

\[ f'(x) = 2x + 3 \]
\[ f'(2) = 2(2) + 3 = 7 \]

b. \( g(x) = x^3 - 2x^2 + 1 \) at \( x = -1 \)

\[ g'(x) = 3x^2 - 4x \]
\[ g'(-1) = 3(-1)^2 - 4(-1) = 3 + 4 = 7 \]

3. Find the equation of the tangent line at the given point.

a. \( f(x) = x^2 + 3x - 4 \) at \( x = 2 \)

\[ f'(x) = 2x + 3 \]
\[ f'(2) = 7 \]
\[ f(2) = 2^2 + 3(2) - 4 = 4 + 6 - 4 = 6 \]

Equation of the tangent line:
\[ y - 6 = 7(x - 2) \]

b. \( g(x) = x^3 - 2x^2 + 1 \) at \( x = -1 \)

\[ g'(x) = 3x^2 - 4x \]
\[ g'(-1) = 7 \]
\[ g(-1) = (-1)^3 - 2(-1)^2 + 1 = -1 - 2 + 1 = -2 \]

Equation of the tangent line:
\[ y + 2 = 7(x + 1) \]

4. Differentiate the given equation.

a. \( f(x) = x^3 - 2x^2 + 1 \)

\[ f'(x) = 3x^2 - 4x \]

b. \( g(x) = \frac{1}{x} \)

\[ g'(x) = -\frac{1}{x^2} \]

5. Sketch the graph of the given equation.

a. \( f(x) = x^2 + 3x - 4 \)

Graph of \( f(x) = x^2 + 3x - 4 \) with key features:
- Vertex: \( (-1.5, -5.25) \)
- x-intercepts: \( x = 1 \) and \( x = -4 \)
- y-intercept: \( y = -4 \)

b. \( g(x) = \frac{1}{x} \)

Graph of \( g(x) = \frac{1}{x} \) with key features:
- Asymptotes: \( x = 0 \) and \( y = 0 \)
- Passes through \( (1, 1) \) and \( (2, 0.5) \)