Assignment 3

Due on 2020-10-15, 23:59:59 IST.

1. List the rules for implementing a function using five isomorphisms. List the common errors that are common mistakes when implementing a function using these isomorphisms. Explain the importance of these rules. (5 marks)

2. Prove the following theorem: If a function f: A → B is bijective, then for every element y in B, there exists exactly one element x in A such that f(x) = y. (5 marks)

3. Define the concept of a group homomorphism. Prove that if f: G1 → G2 is a group homomorphism, then f(1G) = 1G2, where 1G and 1G2 denote the identity elements of G1 and G2 respectively. (5 marks)

4. Let f: R → R be a function defined by f(x) = x^2. Prove that f is continuous at x = 1. (5 marks)

5. Consider the function f(x) = 1/x for x ≠ 0 and f(0) = 0. Determine the domain of f and prove that f is continuous at x = 0. (5 marks)

6. Suppose we have a function f: A → B and two functions g, h: B → C. Let g ∘ f: A → C be the composition of g and f. Prove that if g and h are both continuous, then the composition g ∘ f is also continuous. (5 marks)

7. Let f: [a, b] → R be a continuous function. Prove that f has at least one zero in the interval [a, b] if f(a) and f(b) have opposite signs. (5 marks)

8. Suppose we have a sequence {a_n} in R such that a_n → a as n → ∞. Prove that if f: R → R is continuous at a, then the sequence {f(a_n)} converges to f(a) as n → ∞. (5 marks)

9. Let f: [a, b] → R be a continuous function. Prove that the Riemann integral of f over [a, b] exists. (5 marks)

10. Consider a function f: [a, b] → R that is continuous on [a, b] and differentiable on (a, b). Let c be a point in (a, b) such that f'(c) = 0. Prove that if f has a local minimum at c, then f''(c) ≤ 0. (5 marks)