Assignment 7

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Due on: 2023-04-16, 23:59 UTC

The purpose of this assignment is to practice concepts from the course on partial differential equations.

1. Solve the following boundary value problem:

\[ \begin{align*}
    u(x,0) &= 0, \quad 0 < x < 1 \\
    u(x,1) &= f(x), \quad 0 < x < 1 \\
    u(0,t) &= 0, \quad t > 0 \\
    u(1,t) &= 0, \quad t > 0
\end{align*} \]

(a) Use the method of separation of variables to find a solution.

(b) Discuss the convergence of the series solution.

2. Consider the initial-boundary value problem for the diffusion equation:

\[ \begin{align*}
    u_t &= k u_{xx}, \quad 0 < x < 1, \quad t > 0 \\
    u(0,t) &= 0, \quad t > 0 \\
    u(1,t) &= 0, \quad t > 0 \\
    u(x,0) &= f(x), \quad 0 < x < 1
\end{align*} \]

(a) Find the solution using the method of eigenfunction expansion.

(b) Discuss the physical significance of the solution.

3. Consider the following partial differential equation:

\[ u_{tt} = u_{xx}, \quad 0 < x < 1, \quad t > 0 \]

(a) Find the general solution using the method of separation of variables.

(b) Discuss the stability of the solution.

4. Consider the following partial differential equation:

\[ u_{tt} = u_{xx} + u_{yy}, \quad 0 < x < 1, \quad 0 < y < 1, \quad t > 0 \]

(a) Find the general solution using the method of separation of variables.

(b) Discuss the physical significance of the solution.

5. Consider the following partial differential equation:

\[ u_{tt} = u_{xx}, \quad 0 < x < 1, \quad t > 0 \]

(a) Find the solution using the method of eigenfunction expansion.

(b) Discuss the convergence of the series solution.

6. Consider the following initial-boundary value problem for the heat equation:

\[ u_t = k u_{xx}, \quad 0 < x < 1, \quad t > 0 \]

(a) Find the solution using the method of separation of variables.

(b) Discuss the stability of the solution.

7. Consider the following partial differential equation:

\[ u_{xx} + u_{yyyy} = 0, \quad 0 < x < 1, \quad 0 < y < 1 \]

(a) Find the general solution using the method of separation of variables.

(b) Discuss the physical significance of the solution.