Assignment 10

You may go back to the assignment at any time.

1. Assume your work depends on the normality for the variable Y. Answer the following questions:
   a. Given that $Y$ is normally distributed with mean $\mu$ and standard deviation $\sigma$, find the probability $P(Y > 2)$.
   b. Determine the $95\%$ confidence interval for the mean if you have a sample of size $n = 30$.
   c. If you have a sample of size $n = 50$, what is the probability that the sample mean is within $1$ standard deviation of the population mean?

2. Consider a random variable $X$ with probability density function $f(x) = \begin{cases} \frac{1}{2} & \text{if } 0 < x < 1 \\ 0 & \text{otherwise} \end{cases}$. Find the cumulative distribution function (CDF) $F_X(x)$ for $X$.

3. A random variable $X$ has the following probability mass function (PMF):
   - $P(X = 1) = 0.3$
   - $P(X = 2) = 0.5$
   - $P(X = 3) = 0.2$
   - $P(X = 4) = 0.1$
   Find the expected value $E(X)$ and the variance $\text{Var}(X)$.

4. The amount of time $T$ a customer spends in a store is normally distributed with mean $\mu = 15$ minutes and standard deviation $\sigma = 3$ minutes. Find the probability that a customer spends more than 20 minutes in the store.

5. Consider a random variable $Y$ with the following cumulative distribution function (CDF):
   - $F_Y(-1) = 0$
   - $F_Y(0) = 0.3$
   - $F_Y(1) = 0.8$
   - $F_Y(2) = 1$
   Find the probability that $Y$ is less than 1.

6. A random variable $Z$ has the following probability density function (PDF):
   - $f_Z(z) = \begin{cases} 2z & \text{if } 0 < z < 1 \\ 0 & \text{otherwise} \end{cases}$
   Find the mean $E(Z)$ and the variance $\text{Var}(Z)$.

7. A random variable $W$ follows a Poisson distribution with parameter $\lambda = 3$. Find the probability that $W$ is equal to 4.

8. The following table gives the frequency distribution of the number of accidents per week for a company:
<table>
<thead>
<tr>
<th>Number of Accidents</th>
<th>Frequency</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>5</td>
</tr>
<tr>
<td>1</td>
<td>10</td>
</tr>
<tr>
<td>2</td>
<td>15</td>
</tr>
<tr>
<td>3</td>
<td>20</td>
</tr>
</tbody>
</table>
   Find the expected number of accidents per week.

9. A random variable $X$ follows a Chi-Square distribution with $k = 4$ degrees of freedom. Find the expected value $E(X)$ and the variance $\text{Var}(X)$.

10. A random variable $Y$ follows a Beta distribution with parameters $\alpha = 2$ and $\beta = 3$. Find the expected value $E(Y)$ and the variance $\text{Var}(Y)$.