Assignment 7

1. Let \( A \) be a square matrix of size \( n \times n \). Define \( B = (I_n + A)^{-1} \), where \( I_n \) is the identity matrix. Consider the function \( f(A) = (I + A)^{-1} \).

2. Find the matrix \( B \) such that \( (I_n + B)^{-1} = B \).

3. Consider the system of linear equations \( Ax = b \). Suppose \( A \) is a nonsingular matrix of size \( m \times n \). Let \( x \) be the solution to this system. Show that \( A^{-1}x = b \).

4. Let \( A \) be a square matrix of size \( n \times n \) and \( b \) be a column vector of size \( n \times 1 \). Consider the system of linear equations \( Ax = b \). Show that if \( A \) has a unique solution, then there exists a matrix \( C \) such that \( A = CB \).

5. Let \( A \) be a square matrix of size \( n \times n \) and \( b \) be a column vector of size \( n \times 1 \). Consider the system of linear equations \( Ax = b \). Show that if \( A \) has a unique solution, then there exists a matrix \( C \) such that \( A = CB \).

6. Let \( A \) be a square matrix of size \( n \times n \) and \( b \) be a column vector of size \( n \times 1 \). Consider the system of linear equations \( Ax = b \). Show that if \( A \) has a unique solution, then there exists a matrix \( C \) such that \( A = CB \).

7. Let \( A \) be a square matrix of size \( n \times n \) and \( b \) be a column vector of size \( n \times 1 \). Consider the system of linear equations \( Ax = b \). Show that if \( A \) has a unique solution, then there exists a matrix \( C \) such that \( A = CB \).

8. Let \( A \) be a square matrix of size \( n \times n \) and \( b \) be a column vector of size \( n \times 1 \). Consider the system of linear equations \( Ax = b \). Show that if \( A \) has a unique solution, then there exists a matrix \( C \) such that \( A = CB \).

9. Let \( A \) be a square matrix of size \( n \times n \) and \( b \) be a column vector of size \( n \times 1 \). Consider the system of linear equations \( Ax = b \). Show that if \( A \) has a unique solution, then there exists a matrix \( C \) such that \( A = CB \).

10. Let \( A \) be a square matrix of size \( n \times n \) and \( b \) be a column vector of size \( n \times 1 \). Consider the system of linear equations \( Ax = b \). Show that if \( A \) has a unique solution, then there exists a matrix \( C \) such that \( A = CB \).