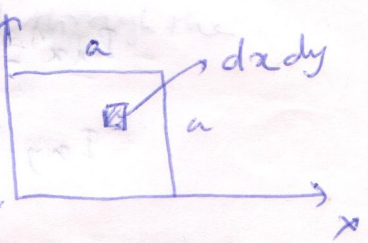
Week-7 – Assignment-7 Solution

① $I_{xx} = \int_{x=0}^{a}\int_{y=0}^{a} (\sigma\, dx\,dy)\, y^2$, σ = mass per unit area.



$= \sigma \int_0^a dx \int_0^a y^2\, dy$ $\quad = M/a^2$

$= \sigma \cdot a \cdot \frac{a^3}{3} = \frac{1}{3}\sigma a^4 = \frac{1}{3}\cdot\frac{M}{a^2}\cdot a^4$

⇒ $\boxed{I_{xx} = \frac{1}{3} M a^2}$

Similarly, $I_{yy} = \int_{x=0}^{a}\int_{y=0}^{a} (\sigma\, dx\,dy)\, x^2 = \frac{1}{3} M a^2$

② $I_{zz} = \int_{x=0}^{a}\int_{y=0}^{a} (\sigma\, dx\,dy)\,(x^2+y^2)$

$= \int_{x=0}^{a}\int_{y=0}^{a} \sigma\, dx\,dy \cdot x^2 + \int_{x=0}^{a}\int_{y=0}^{a} \sigma\, dx\,dy \cdot y^2$

$= \frac{1}{3} M a^2 + \frac{1}{3} M a^2 = \frac{2}{3} M a^2$

③ I_{xy} = Product of inertia of the plate about x and y axes

$= -\int_{x=0}^{a}\int_{y=0}^{a} (\sigma\, dx\,dy)\, xy = -\sigma \left(\frac{x^2}{2}\right)_0^a \left(\frac{y^2}{2}\right)_0^a$

$= -\sigma \cdot \frac{a^2}{2}\cdot\frac{a^2}{2} = -\frac{M}{a^2}\cdot\frac{a^4}{4} = -\frac{1}{4} M a^2$

Since, distance of element dxdy from yz, xz and xy planes are, x, y and 0 respectively.

$\therefore I_{xz} = I_{zx} = 0$ and $I_{yz} = I_{zy} = 0$