1. 

\[ y' = x^2 + y^2 \]
\[ y'' = 2x + 2yy' \]
\[ y''' = 2 + 2(y')^2 + 2yy' \]
\[ y'''' = 6y'y'' + 2yy'' \]

\[ y_1 = 0.002667 \text{ at } x = 0.2 \]

\[ y_2 = 0.021333 \text{ at } x = 0.4 \]

2. \( h = 0.5, \ y(1) = 2; \ y(1.5) = 2 + 0.5[3(1)^2 + 1] = 4 \)

\[ y(2) = 4 + 0.5[3(1.5)^2 + 1.5] = 7.875 \]

\[ x_0 = 1, \ y_0 = 2; \ y_1 = y(1.5) = 4 \ (y'' = 6x, \ y''' = 6) \]

Truncation error

Step 1:

\[ E_{t,1} = \frac{y''_0}{2} h^2 + \frac{y'''_0}{6} = \frac{6(1)}{2} (0.5)^2 + \frac{6}{6} (0.5)^3 = 0.875 \]

Step 2:

\[ x_1 = 1.5, y_1 = 4.0, y_2 = y(2.0) = 7.875 \]

\[ E_{t,2} = \frac{6(1.5)}{2} (0.5)^2 + \frac{6}{6} (0.5)^3 = 1.25 \]

3. The analytic solution yields,

\[ y = -\frac{1}{2} x^4 + 4x^3 - 10x^2 + 8.5x + 1 \]

\[ y(1) = 3 \]

Using Euler method (with \( h = 0.5 \)), the solution is,

\[ y(1) = 5.875 \]

The percent relative error is,

\[ \epsilon = \frac{3 - 5.875}{3} \times 100\% = 95.8\% \]

Again using Euler method (with \( h = 0.25 \)),

\[ y(1) = 4.3438 \]

The percent relative error is,

\[ \epsilon = \frac{3 - 4.3438}{3} \times 100\% = 44.79\% \]
4. Analytic solution,
\[ y(x) = e^{x^3} - 1.2x \]

\[ h = 0.5, \quad f(x) = x^2y - 1.2y, \]
\[ y_{i+1} = y_i + f(x_i, y_i)h. \]
\[ y_{i+1} = y_i + \frac{h}{2}[f(x_i, y_i) + f(x_{i+1}, y_{i+1}^{k-1})] \]

\[ i = 0, \text{ Predictor:} \]
\[ y_1^0 = 1 + (1 \times 0^2 - 1.2 \times 1)0.5 = 0.4 \]
\[ \text{Corrector:} \]
\[ k = 1, \quad y_1^1 = 0.605, \quad \epsilon = 33.9\% \]
\[ k = 2, \quad y_1^2 = 0.5563121, \quad \epsilon = 8.75\% \]
\[ k = 3, \quad y_1^3 = 0.5678757, \quad \epsilon = 2.04\% \]
\[ k = 4, \quad y_1^4 = 0.5651295, \quad \epsilon = 0.49\% \]

Thus, \( y_1 = y(0.5) = 0.5651295 \)

\[ i = 1, \text{ Predictor:} \]
\[ y_2^0 = 0.2966930 \]
\[ \text{Corrector:} \]
\[ k = 1, \quad y_2^1 = 0.4160766, \quad \epsilon = 28.7\% \]

continuing like this
\[ y(1) = 0.4104059 \]
\[ y(1.5) = 0.5279021 \]
\[ y(2) = 2.181574 \]

5. \( h = 0.5 \)

\[ i = 0, \text{ Predictor:} \]
\[ y_{0+\frac{1}{2}} = 1 + (1 \times 0^2 - 1.2 \times 1)0.25 = 0.7 \]
\[ \text{Corrector:} \]
\[ y_1 = y(0.5) = 1 + (0.7 \times 0.25^2 - 1.2 \times 0.7)0.5 = 0.601875 \]

\[ i = 1, \text{ Predictor:} \]
\[ y_{1+\frac{1}{2}} = 0.4589297 \]
\[ \text{Corrector:} \]
\[ y_2 = y(1) = 0.4555911 \]

\[ i = 2, \text{ Predictor:} \]
\[ y_{2+\frac{1}{2}} = 0.4328117 \]
\[ \text{Corrector:} \]
\[ y_3 = y(1.5) = 0.5340383 \]

\[ i = 3, \text{ Predictor:} \]
\[ y_{3+\frac{1}{2}} = 0.6742233 \]
\[ \text{Corrector:} \]
\[ y_4 = y(2) = 1.1619087 \]