1. Obtain the Hamiltonian for a charged particle in a time varying electromagnetic field. Write it explicitly by neglecting terms in square of the magnetic field, \( \mathbf{B} \), \( \mathcal{O}(\mathbf{B}^2) \) (or rather \( \mathcal{O}(\mathbf{A}^2) \), where \( \mathbf{A} \) is the magnetic vector potential).

2. Describe the motion of a charged particle in a constant magnetic field, \( \mathbf{B} = B\mathbf{k} \). Choose the vector potential, \( \mathbf{A} = (-B, 0, 0) \) and a gauge in which the scalar potential, \( \phi = 0 \). This is called as the Landau gauge.

3. A particle of charge \( q \) executing simple harmonic motion along \( x \)-axis is acted upon by a time dependent electric field having a form, \( E = E_0 e^{-t^2/\tau^2} \), where \( E_0 \) and \( \tau \) are constants. If the oscillator is in the ground state at \( t = -\infty \), using first order time dependent perturbation theory, find the probability that the particle will be found in one of the excited states at \( t \to \infty \). Also explain which of the excited state(s) it can go to?

4. Compute the coefficients \( c_m(t) \) and \( c_k(t) \) upto second order for a two-level system for a general initial condition \( c_m(0) = m \) and \( c_k(0) = k \).

5. Find a relation between the half-life \( (t_{1/2}) \) of an excited state and the lifetime \( (\tau) \) of the state.

6. Consider a perturbation of the form applied to a two-level system \( (a \) and \( b) \),
\[ H' = U\delta(t) \]
Assume \( U \) is Hermitian and the diagonal elements of \( U \) between the unperturbed states of the two-level system vanishes. Further assume \( c_a(-\infty) = 1 \) and \( c_b(-\infty) = 0 \). Find \( c_a(t) \) and \( c_b(t) \). Also find the probability of transition as \( t \to \infty \).
Assume:
\[
\delta_\epsilon(t) = \begin{cases} 
\frac{1}{2\epsilon} & \text{for } -\epsilon < t < \epsilon \\
0 & \text{otherwise}
\end{cases}
\]