Assignment 3

Due on 2000-01-15, 03:00 ET.

1. Find the eigenvalues and eigenvectors of the following matrix:
   \[
   A = \begin{pmatrix}
   1 & 3 \\
   2 & 4 \\
   \end{pmatrix}
   \]

2. Consider the eigenvalue equation \( Au = \lambda u \) where \( u \) is a non-zero vector and \( A \) is a matrix. Prove that if \( \lambda \) is an eigenvalue of \( A \), then \( \lambda^2 \) is also an eigenvalue of \( A^2 \).

3. Let \( T \) be a linear operator on a finite-dimensional vector space. Prove that if \( T \) is diagonalizable, then \( T^2 \) is also diagonalizable.

4. Consider the operator \( T : \mathbb{C}^2 \to \mathbb{C}^2 \) defined by \( T(a, b) = (a, 2b) \). Is \( T \) a normal operator? Justify your answer.

5. Let \( H = L^2([0,1]) \) be the Hilbert space of square-integrable functions on \([0,1]\). Define the operator \( A \) on \( H \) by \( \langle Au, v \rangle = \int_0^1 u'(t)v(t) \, dt \) for all \( u, v \in H \). Is \( A \) a normal operator? Justify your answer.