NPTEL course offered by IIT Madras
Computer methods of analysis of offshore structures

Tutorial 11:Random vibration

Answer all questions **Total marks: 25**

1. Explain a stationary and Ergodic process

- Offshore structures are exposed to various Environmental loads -

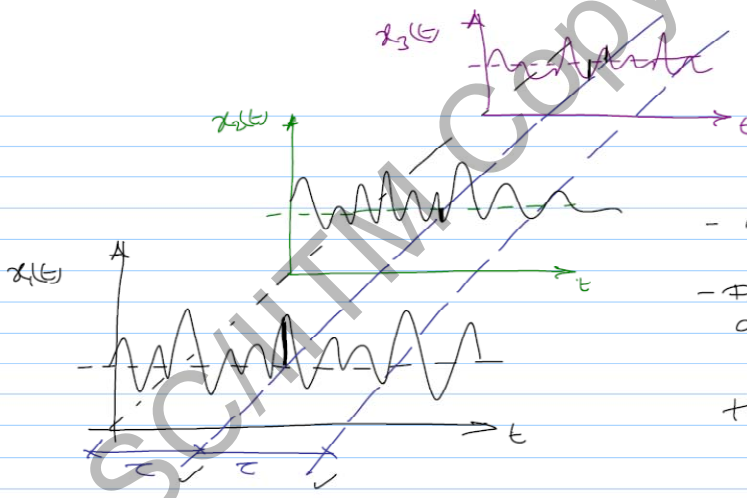
piece-wise stationary process

Stationary process is a process for which

statistical properties like mean value, standard deviation

are same for all points

in both time and position



Draw lines across the random process @ intervals of τ

- Find mean square value (MSV)
- If MSV is unique across the sample, (ensemble)

then process is called stationary process

Ergodic process – special stationary process

- Let us take about 100 samples
- find MSV along the time line
- This MSV is equal to the unique MSV of the ensemble, then the process is called Ergodic process.

Ergodic process is process representing a single sample is the ensemble which is (arbitrarily) chosen and has same MSV as that of the unique MSV of the ensemble.

2. Define Ergodic process, mathematically

- Constant mean of any process : $\mu_x = E[x(t)]$ —(1)

auto-covariance is given by

$$\Gamma_x(\tau) = E\left[(x(t) - \mu_x)(x(t+\tau) - \mu_x)\right] \quad —(2)$$

which should depend only on the interval (τ), and not on time t

- time independent.

μ_x and $\Gamma_x(\tau)$ are not time averages – Ensemble averages

process can be stated as mean-Ergodic, if the time-average estimate also converges to mean value of the ensemble average

$$\hat{\mu}_x = \frac{1}{T}\int_0^T x(t)\,dt \longrightarrow \text{converges to } \mu_x \text{ @ } T \to \infty$$

Then the process is said to be auto-covariance ergodic if time-average converges to the ensemble value

$$\hat{\gamma}_x(\tau) = \frac{1}{T}\int_0^T [x(t+\tau) - \mu_x][x(t) - \mu_x]\,dt$$

$\Rightarrow$ converges to $\gamma_x(\tau)$ as $T \to \infty$

Example of Ergodic process

Stationary Gaussian process ✓

In case of discrete random process, Ergodicity can be verified:

Let $X(n)$ is ergodic, which represents a discrete-time random process,

if mean converges to the ensemble average

$$\mu_x = \frac{1}{N}\sum_{n=1}^{N} x[n] \Rightarrow \text{Converges to ensemble average } E[X] \text{ @ } N \to \infty,$$

then ergodicity is confirmed

3. Derive transfer function, connecting response spectrum and load spectrum
4. Show that transfer function is proportional to Dynamic amplification factor
5. Define white noise approximation