NPTEL course offered by IIT Madras

Risk and Reliability of Offshore structures

<u>Tutorial 2: Rules of Probability</u>

Answer all questions **Total marks: 25**

1. **Explain principles of plausible reasoning and their use in Reliability analysis**

Rules of thinking are generally termed as plausible reasoning. Probability theories are complex in nature and one finds it difficult to learn as it tries to model everything that a human brain shall think. However, system for probability, based on plausible reasoning looks like abstract in the beginning but everything becomes derivable. This leaves no chance for confusion. Probability theory is nothing but common sense reduced to calculation (Laplace, 1819). Probabilistic analysis has two classes namely: i) Bayesian class; and ii) frequency class. In Bayesian class, prior information of the subject is included, which improves the knowledge status of the unknown. Derived information is based on the available data, called posterior information. Frequency class is based on the sampling distributions. This is not capable of incorporating the prior information. It further assumes that all the realizations with the sample are independent.

2. **Explain deductive logic, with an example**

Let us consider a simple example to illustrate deductive logic:
On a late night, policemen were doing patrolling duty on a main road of a city centre. They heard a loud sound similar to that of breaking of a glass. They also noticed a man running in the dark, wearing a mask on his face. The man was also carrying a big bag in his hand and the policemen would think that he is a robber. The incident also has other similar options as follows:
Policemen did not know complete information about the incident. The person who was noticed to be running in the dark can also be the owner of the store who was coming out of the shop. Instantly, there was a loud sound being heard, which was similar to that of breaking of a glass. However the fact remains verified that the shop window was found broken, which must have been damaged earlier. The

man who was walking by the side of the shop with a bag was misunderstood as a thief. While policemen decided that he was a thief, he could also be the owner of the shop, who is returning home after closing the shop. Now, let us examine the reasons for policemen thinking that the man as a thief. One of the main reasons could be due to the past (similar) experience making plausibility to think the man as the thief.

Therefore, following propositions hold good:

A: Window glass broken; gentle man with mask; sneaking out; having a big bag in his hand; dark night

B: gentlemen is the thief

Given B is true $\Rightarrow$A is more plausible – this is a direct problem.

What policemen see is assigned as A, which is true; Seen A is true, they decided that the man is a thief. In fact, seen A is true, B becomes more plausible, which is an inverse problem. If A is true, then B is true: This is a Hypothesis. Considering the example cited above, it is seen that the policemen observed that A is true. If this is established, which means that B is true. This is the famous A $\Rightarrow$B statement, which is deductive logic. This is deduced based on various factors like visual observation, experience, correlation of facts etc. This also means that if B is false, A is false. This statement confirms that there is only a logical dependence and no physical dependence.

3. Explain how deductive reasoning is important to quantify deductive logics?

Considering the same example as explained above, following statements are valid:

If A is true, then B is true. It is also interesting (and important) to note that A is true and therefore B is true. However such deductive logics cannot be readily applied to examine reliability of the offshore structures due to higher order of uncertainties. In reality, such analysis deals (or forced to deal with) weaker reasoning. In such cases, deductive reasoning shall become more important. Let us consider the following example:

A: it will start to rain by 10 am at the latest

B: the sky will become cloudy before 10 am

If A is true, then B is true; for B is true, A becomes more plausible. If B is false, then A becomes less plausible.

The above events (statements) show that if A is true, B expresses only as the logical consequence of A and not a causal physical consequence; this leads to a weaker reasoning. Alternatively, if A is true, then B becomes more plausible. If B is true, A becomes more plausible. Therefore, in plausible reasoning, judgement is not only to decide whether something becomes more (or less) plausible but also evaluate the degree of its plausibility in some manner. Referring to the above example, plausibility of rain by 10 am strongly depends on the darkness of the clouds at 9.45. Hence, in deductive reasoning, one is very much dependent on the prior information in order to decide the degree of plausibility. This reasoning process takes place almost in the sub-conscious state of the judgement.

4. State and explain Product rule of plausible reasoning

Product rule

It is necessary to seek a consistent rule relating the plausibility of AB|C to those of A|C and B|C separately. The process of deciding that AB is true can be broken down into two parts: Decide that B is true B|C. Having accepted that B is true, decide that A is true A|BC. Or equivalently, decide that A is true A|C; having accepted that A is true decide that is true B|AC. Formally we can state this as follows:

$AB|C = F[B|C, A|BC]$

The rule of agreement implies that given any change in prior information, such that B becomes more plausible but A does not change. This can be expressed as:

$B|C'' > B|C$

$A|BC = A|BC''$

By simple observation, one can state that AB becomes only more plausible and not lesser. under the above conditions.

$A|BC'' = A|BC$

Introducing the real number $X = B|C, y = A|BC$, the function F can be written as F(x, Y). This results in F (X,Y) being a continuous and monotonic increasing function of X and Y. F(x, y) has to be continuous to prevent a large increase in the plausibility of AB due to a small increase of plausibility of A|C or B|C. This implies the following:

$F_1(x, y) = \frac{\partial F}{\partial x}$

$F_2(x, y) = \frac{\partial F}{\partial y}$

Where, F_i denotes differentiation with respect to the i^{th} argument of F.

For example, one is interested to know the plausibility (ABC-D) that the three statements are simultaneously true because of the fact that Boolean algebra is associative. This can be evaluated in two ways. One way is to consider BC as a single statement and is given by:

$ABC|D = F[BC|D, A|BCD] = F\{F[C|D, B|CD], A|BCD\}$

Alternatively, AB is considered as a single statement and is expressed as:

$ABC|D = F[C|D, AB|CD] = F\{C|D, F[A|BCD, B|CD]\}$

In such case, following statement hold good:

$F[F(x, y), Z] = F[x, F(z, y)]$

Above equation is known also as the *Associativity equation*. It is evident that the above equation has a trivial solution, that is F(x, y) is constant. However, as this solution violates the monotonic requirement, it is of no use. By using the following abbreviations, we get:

$u \equiv F(x, y)$

$v \equiv F(y, z)$

Therefore, Eq. (1.48) will be reduced to the following form:

$F(u, z) = F(x, v)$

Differentiating the above equation with respect to x and y, we obtain as follows:

$F_1(u, z)F_1(x, y) = F_1(x, v)F_1(u, z)F_2(x, y) = F_2(x, v)F_2(y, z)$

This leads to the following statement:

$\frac{F_2(x,v)F_1(u,z)}{F_1(x,y)} = \frac{F_1(x,v)}{F_2(x,y)}$

Defining the notation, $\mathrm{G}(\mathrm{x}, \mathrm{y}) \equiv \frac{F_2(x,y)}{F_1(x,y)}$ we can write the above equation as follows:

$u = G(x, v)F_1(y, z) = G(x, y)$

The above equation can be re-written as:

$V = G(x, v)F_2(y, z) = G(y, Z)G(x, y)$

Denoting the left hand sides of above equations by U and V respectively, one can write as follows:

$$\frac{\partial u}{\partial z} = \frac{\partial G(x,v)F_1(y,z)}{\partial z} = \frac{\partial G(x,y)}{\partial z} = 0$$

$$\frac{\partial u}{\partial z} = \frac{\partial G(x,v)\partial F(y,z)}{\partial y \partial z} = \frac{\partial G(x,v)F_2(y,z)}{\partial y} = \frac{\partial v}{\partial y}$$

This implies that V = G(x, y)G(y ,z) is independent of Y. The most general function G(x, y) with this property is given by:

$$G(x,y) = r\frac{H(v)}{H(y)}$$

Where, r is a constant and H(x) is arbitrary. Since F is a monotonic function, for G>0 it is required that also R>0. Based on the above equations, one can arrive as follows:

$$F_1(y,z) = \frac{H(v)}{H(y)} F_2(y,z) = r\frac{H(v)}{H(z)}$$

Therefore, the relation $\mathrm{dv} = \mathrm{dF}(\mathrm{y}, \mathrm{Z}) = \mathrm{F}_1\mathrm{dy} + \mathrm{f}_2\mathrm{dz}$ takes the form:

$$\frac{dv}{H(v)} = \frac{dy}{H(y)} = \frac{dz}{H(z)}$$

It can be shown that a non-trivial solution for the above equation is in the following form:

$$w[F(x,y)] = w(x)w(y)$$

By introducing x=B|C and y=A|BC, above equation can be re-written as:

$$p(AB|C) = p(B|C)p(A|BC)$$

Above equation is termed as *Product Rule*. By its construction, it is seen that P (.) should be a positive, continuous, monotonic function, which can be either increasing or decreasing. Now let us consider the limiting cases. First one is the case in which A|C is certain, satisfying the following condition:

AB|C=B|C and A|BC=A|C are true.

By expanding the above relationship using Product Rule, we get:

P(A|BC)p(B|C)=p(A|C)p(B|C)=p(B|C)

The above equation results in p(A|C)=1, which is certainty. Alternatively, a case corresponds to A|C is impossible in which the following condition is necessary:

AB|C=B|C and also A|BC=A|C

By expanding the above using Product Rule, we get:

p(A|C)p(B|C)=P(B|C)

This holds for two values of p(A|C) ,0 and +α. If we choose the solution p(A|C)=0 as a convention, this results in $0 \le p(x) \le 1$

5. State and explain Sum Rule of plausible reasoning

Sum rule

Let the plausibility of $\acute{A}$ be related to the plausibility of A. It can be easily shown that the functional form is given by:

$p(A|B) + p(\acute{A}|B) = 1$

It is interesting to assess whether these set of rules are adequate to decide the plausibility of any logic function $f(A_1, A_2 \ldots\ldots A_n)$ of propositions $\{A_1, A_2 \ldots\ldots A_n\}$?. Let us seek a general formula for the logical sum A+B by applying repeatedly the product and sum rules as explained below:

$$p(A+B|C) = 1 - p(\acute{A}\acute{B}|C)$$
$$= 1 - p(\acute{A}|C)p(\acute{B}|\acute{A}C)$$
$$= 1 - p(A\acute{|}C)[1 - p(\acute{B}|\acute{A}C)]$$
$$= 1 - p(\acute{A}|C) + p(\acute{A}|C)p(\acute{B}|\acute{A}C)$$
$$= p(A|C) + p(\acute{A}|BC)p(B|C)$$
$$= p(A|C) + [1 - p(A|BC)]p(B|C)$$
$$= p(A|C) + p(B|C) - p(B|C)p(A|BC)$$
$$= p(A|C) + p(B|C) - P(AB|C)$$

The generalized sum rule is one of the most useful relationships.