Introduction to crystal elasticity and crystal plasticity

Module 5: Hardening mechanism in metals

1. The strength of metals increases due to
(a) increase in grain size
(b) decrease in number of dislocations
(c) presence of solute atoms
(d) strain-softening
Ans : (c)

2. The solute atoms provides the hardening effect more in cases of
(a) interstitial solid solution
(b) substitutional solid solution
(c) pure metals
(d) none of these
Ans : (a)

3. In dispersion strengthening, the strength increases
(a) with decreasing volume fraction of particles
(b) with increasing particle fraction
(c) with increasing volume fraction of particles
(d) none of the above
Ans : (c)

4. Which of the following statements is correct?
(a) Commercial metal do not deform according to the laws of single crystal
(b) Strength is inversely proportion to dislocation mobility
(c) Plastic deformation of single crystal occurs by the movement of dislocations
(d) Upper yield point is generally reported since it is more reproducible
Ans : (d)

5. In which of the following crystal strain hardened rapidy?
(a) BCC   (b) HCP   (c) FCC   (d)SC
Ans : (c)

1. Consider a precipitation-strengthened aluminum alloy. After an appropriate heat treatment, the microstructure of the alloy consists of precipitates with a mean spacing of 0.2 µm. Compute the shear stress required for Orowan bowing of dislocations in this material. Aluminium is having FCC structure and lattice parameter is 0.4 nm.
(a)73.32 Mpa
(b)75.32 Mpa
(c) 78.32 Mpa
(d) 71.32 Mpa
Ans : (a)

Explanation: For bow out of dislocation in FCC Al-

\[ \tau = \frac{2Gb^2}{bd} = \frac{2Gb}{d} \]

Where, \( G = 26 \) GPa, \( d = 0.2 \times 10^{-6} m \)
\( b = 0.282 \) nm \( (a/\sqrt{2}) \)

\( \tau = \frac{2\times26\times10^9\times0.282\times10^{-9}}{0.2\times10^{-6}} = 73320000 \frac{N}{m^2} = 73.32 \) Mpa

2. Consider a dispersion-strengthened alloy with average interparticle spacing of \( \lambda \). If \( N_v \) is the number of particles per unit volume, \( d \) is the mean particle diameter, and \( f \) is the volume fraction of particles, then what will be the expression for interparticle spacing in terms of mean dia. and volume fraction .

(a) \( \lambda = d\left[(1/2 f)^{1/3} - 1\right] \)
(b) \( \lambda = d\left[(1/2 f)^{1/8} - 1\right] \)
(c) \( \lambda = d\left[(1/2 f)^{1/4} - 1\right] \)
(d) \( \lambda = d\left[(3/2 f)^{1/2} - 1\right] \)

Ans : (a)

Explanation: \( f = \frac{(\frac{4}{3})\pi r^3 N_v}{1} \)

\( \Rightarrow f = \frac{4}{3} \pi \left(\frac{d}{2}\right)^3 N_v \)

\( \Rightarrow d = \left(\frac{3f}{2\pi N_v}\right)^{\frac{1}{3}} \)

Again, for one particle \( = \frac{(4/3)\pi r^3 \times 1}{D^3} \)

\( = \frac{(\pi/6)d^3}{(\lambda+d)^3} \)

\( \Rightarrow \frac{\lambda+d}{d} = \left(\frac{\pi}{6f}\right)^{\frac{1}{3}} \)

\( \Rightarrow 1 + \frac{\lambda}{d} = \left(\frac{\pi}{6f}\right)^{\frac{1}{3}} \approx \left(\frac{1}{2f}\right)^{\frac{1}{3}} \)
\[ \Rightarrow \lambda = d \left[ \left( \frac{1}{2\pi} \right)^{\frac{3}{2}} - 1 \right] \]

3. For a rod of an fcc metal with a fiber texture, what is the maximum amount of texture strengthening possible? (what is the highest possible ratio of yield strengths for textured material to those of randomly oriented material?) what is the greatest amount of texture softening possible?

(a) 3.67, 2.44  (b) 4.56, 3.5  (c) 2.3, 0.5  (d) 3.2, 1.5

Ans: (a)

Explanantion: As we know that,

i) \( M \) (taylor factor) = \( \frac{dy}{d\varepsilon} = \frac{\sigma_x}{\tau} \)

This ratio will tell that maximum amount of texture strengthening possible specified orientation. and as we know that in FCC, [111] is closed pack direction.

So, \( M = 1.5\sqrt{6} = 3.67 \)

ii) The greatest amount of texture softening possible in [100] orientation –

So, \( M = \sqrt{6} = 2.44 \)

4. when music wire (100% pearlite) is cold drawn, the pearlite spacing is reduced in proportion to the wire diameter. That is, \( \frac{d}{d_0} = \frac{D}{D_0} \), where \( d \) is the pearlite spacing and \( D \) is the wire diameter. The \( o \) subscript indicates original conditions. Assuming that the flow stress, \( \sigma \), is given by \( \sigma = \sigma'_o + k' \cdot d^{-1/2} \), derive an expression for \( \sigma \) as a function of strain, \( \varepsilon \).

(a) \( \sigma = \sigma'_o + k' \cdot \left( d_0 e^{\left( -\frac{\varepsilon}{2} \right)} \right)^{-\frac{3}{2}} \)

(b) \( \sigma = 2\sigma'_o + k' \cdot \left( d_0 e^{\left( -\frac{\varepsilon}{2} \right)} \right)^{-\frac{3}{2}} \)

(c) \( \sigma = \sigma'_o + 2k' \cdot \left( d_0 e^{\left( -\frac{\varepsilon}{2} \right)} \right)^{-\frac{1}{2}} \)

(d) \( \sigma = \sigma'_o + k' \cdot \left( d_0 e^{\left( -\frac{\varepsilon}{2} \right)} \right)^{-\frac{1}{2}} \)

Ans: (d)

Explanantion: \( \sigma = \sigma'_o + k' \cdot d^{-1/2} \)
\[ \varepsilon = \ln \frac{A_0}{A} = 2\ln \frac{d_0}{d} \Rightarrow d_0 e^{\left(\frac{-\varepsilon}{2}\right)} = d \]

So, \( \sigma = \sigma_0' + k' \left( d_0 e^{\left(\frac{-\varepsilon}{2}\right)} \right)^{\frac{1}{2}} \)

5. Consider a material strengthened by precipitation of fine particles. Suppose the shear yield strength of the particles is \( G/10 \) (theoretical) and the strength of the matrix is \( G/500 \). Also, assume that the volume fraction of particles is 1 vol% and they have a diameter of 0.010 \( \mu \)m. Assume that \( b = 0.2 \) nm. Calculate the applied shear stress, \( \tau \) (relative to \( G \)), which is necessary to bow the dislocations between particles.

(a) \( 4.98 \times 10^{-3} G \)
(b) \( 2.98 \times 10^{-3} G \)
(c) \( 3.98 \times 10^{-3} G \)
(d) \( 1.98 \times 10^{-3} G \)

Ans: (b)

Explanantion: As, \( \tau = \tau_{matrix} (1 - V_f) + \tau_{particle} V_f \)

\[ = 0.002(1-0.01) G + (0.01 \times 0.1) G \]

\[ \tau = 1.98 \times 10^{-3} G + 0.001 G \]

\[ \therefore \tau = 2.98 \times 10^{-3} G \]

6. Consider aluminium alloy containing \( \text{Al}_2\text{Mg} \) precipitates. (i) Find out the expression of maximum shear stress for particle shear in terms of surface energy and radius of particle, critical spacing between particles and Berger’s vector. (ii) Calculate the critical spacing of precipitates at which the mechanism of hardening changes from particle shear to dislocation bowing. Assume the precipitates are arrayed as simple cubic. Specific surface energy of particle 1400 mJm\(^{-2}\), atomic radius of Al is 0.143 nm and shear modulus 26.1 GPa. Note that Al is having FCC structure.

(a) \( \frac{\pi r_o \gamma}{2 b x} \), 9.7 Å
(b) \( \frac{\pi r_o \gamma}{4 b x} \), 14.7 Å
(c) \( \frac{\pi r_o \gamma}{4 b x} \), 10.7 Å
(d) none of these
Ans : (a)

Explanation:

Force per unit length for the dislocation = \( \tau_{\text{shear}} \times b \)
So, total force on dislocation length = \( \tau_{\text{shear}} \times b \times x \)

Work done during dislocation movement (2r_0) = \( \tau_{\text{shear}} \times b \times x \times 2r_0 \)
And, surface energy at interface due to cutting the particle = \( \pi r_0^2 \gamma \)

Where \( \gamma \) is surface energy per area.

\[
\tau_{\text{shear}} \times b \times x \times 2r_0 = \pi r_0^2 \gamma
\]
\[
\tau_{\text{shear}} = \frac{\pi r_0^2 \gamma}{2bx}
\]

ii) Due to dislocation bowing out:

\[
\tau_{\text{Orowan}} = \tau_{\text{Shear}}
\]

\[
\Rightarrow \frac{\pi r_0 \gamma}{2bx} = \frac{Gb}{x}, \quad [r_0 = r_{Al_2Mg}]
\]

\[
r_{Al_2Mg} = \frac{Gb^2}{\pi \gamma} = 9.7 \text{ Å}
\]

\[
= \frac{2\times26.1\times10^9\times(2\times0.143\times10^{-9})^2}{\pi\times1400\times10^{-3}} = 9.7 \text{ Å}
\]

7. Consider an alloy of Al-Mg having 5% Mg by weight. The solid solution contains \( \alpha \) – phase (1% Mg by weight) and the particle of Al\(_2\)Mg (\( \beta \) – phase of 35% Mg by weight) is precipitate out. Assume density of Al and Al\(_2\)Mg are 2.7 g/cm\(^3\) and 2.3 g/cm\(^3\).

(a) What is the volume fraction of precipitated particles?
(b) Calculate critical spacing between precipitates at which the hardening mechanism changes from dislocation bowing (Orowan strength) to dislocation cut through the particles. Assume that the precipitates are arrayed like simple cubic crystal structure. The shear yield strength of matrix (Al) is \( G_{Al}/500 \) and the particles are \( G_{Al}/10 \) where \( G_{Al} = 26.1 \text{ GPa} \). The atomic radius of Al is 0.143 nm and it follows FCC structure.

(a) 0.14 ; 36.38
(b) 0.12 ; 36.38
(c) 0.10 ; 34.36

Ans : (a)

Explanation: As we know that from lever rule,
\[
\beta \text{ phase fraction} = \frac{\frac{5-1}{35-1}}{\frac{4}{34}} = 0.12
\]

So, \(\alpha\) phase = 0.88

\[
\therefore \text{ Volume fraction} \quad V_f = \frac{(0.12/2.3)}{\frac{0.88}{2.3} \times \frac{0.12}{2.3}} = 0.14
\]

b) \(\tau_{Orowon} = \frac{2Gb}{d}\)

\[
\tau = \tau_{matrix} (1 - V_f) + \tau_{particle} V_f
\]

\[
= \frac{G_{Al}}{500} (1 - 0.14) + \frac{G_{Al}}{10} \times 0.14
\]

\[
= 0.01572 \times G_{Al}
\]

\[
= 0.4103 \text{ GPa}
\]

\[
0.01572 \times G_{Al} = \frac{2Gaib}{d}
\]

\[
d = \frac{2 \times 0.286 \times 10^{-9}}{0.01572} = 36.38 \times 10^{-9} = 36.38 \text{ nm}
\]