Introduction to crystal elasticity and crystal plasticity

Module 4: Crystal Plasticity (Part A)

Syllabus: Crystal defects; Dislocation source; Dislocation motion and slip; Critically resolver shear stress; Dislocation energy

1. The direction of dislocation line is parallel to the Burger’s vector in a dislocation of the type:
   (a) screw  (b) edge  (c) mixed  (d) none of these
   Ans: A

2. The staking arrangement of FCC crystal structure is
   (a) ABCABCABC…  (b) ABABAB…  (c) ABABCABA…  (d) ABCABABC……
   Ans: A

3. The possible slip plane of BCC structure is
   (a) {100}  (b) {111}  (c) {110}  (d) {101}
   Ans: C

4. The stored energy in a copper crystal due to the strain fields of dislocations is 12,000 kJ m\(^{-3}\). The dislocation density of copper is …………
   Assume shear modulus, \(G = 48\) GPa and Burgers vector, \(b = 0.26\) nm.
   
   \(E = \frac{Gb^2}{2}\)  
   \(G = 48\) GPa  
   \(b = 0.26\) nm  
   \(E = 12000\) kJ/m\(^3\)  
   \[\therefore E = \frac{48 \times 10^9 \times (0.26 \times 10^{-9})^2}{2} = 1.622 \times 10^{-9}\] J/m

   \(\rho = \frac{(12000 \times 10^9)}{1.622 \times 10^{-9}} = 7.39 \times 10^{15}\) m\(^{-2}\)

   Ans: E = \(\frac{Gb^2}{2}\)
5. Copper has FCC crystal structure. If the lattice parameter, \( a = 0.36 \) nm, the length of the Burgers vector for the slip plane to operate is 

(a) 0.25 nm   (b) 0.52 nm   (c) 0.21 nm   (d) 0.35 nm 

Ans: 
Cu is FCC 
\[ a = 0.36 \text{ nm} \quad 4r = \sqrt{3}a \] 
\[ b = \frac{0.36}{\sqrt{2}} \text{ nm} \quad r = \frac{\sqrt{3}}{4}a \] 
\[ \therefore b = 0.25 \text{ nm} \quad b = \frac{a}{\sqrt{2}} \]

6. The tilt angle of a tilt boundary in BCC iron (\( a = 0.287 \) nm) with edge dislocations 750 nm apart is 

(a) 0.02°   (b) 0.2°   (c) 0.32°   (d) 0.12° 

Ans: Given : \( a = 0.287 \) nm , 
Dislocation distance (D) = 750 nm 
For BCC structure \( b = (\sqrt{3}/2)a = 0.2485 \) nm 
As we know that, \[ \sin \left( \frac{\theta}{2} \right) = \frac{b}{2D} \] 
So \( \theta = 2 \sin^{-1} \left( \frac{b}{2D} \right) = 2 \sin^{-1} \left( \frac{0.2485}{2 \times 750} \right) = 0.02° \)

7. At \( 750 \) °C, a solid has one vacancy for every \( 10^5 \) atoms. When the temperature is \( 900 \) °C, this number is 1 in \( 10^4 \). The activation energy required to form a vacancy in the solid is 

……………………………………kJ/mol. Consider gas constant \( R = 8.314 \) J/mol-K. 

(a) 15.31 kJ/mol   (b) 31.35 kJ/mol   (c) 11.28 kJ/mol   (d) 21.28 kJ/mol 

Ans: \( R = 8.314 \) J/mol-K 
\[ K = Ae^{\frac{E_a}{RT}} \] 
\[ K_1 = Ae^{\frac{E_a}{RT_1}} \] 
\[ K_2 = Ae^{\frac{E_a}{RT_2}} \] 
\[ \Rightarrow \frac{K_1}{K_2} = e^{\frac{E_a}{RT_1}} e^{-\frac{E_a}{RT_2}} \] 
\[ \Rightarrow \ln \left( \frac{K_1}{K_2} \right) = - \frac{E_a}{RT_1} + \frac{E_a}{RT_2} \]
\[ \ln \left( \frac{K_1}{K_2} \right) = \frac{Ea}{R} \left( \frac{1}{T_2} - \frac{1}{T_1} \right) \]
\[ \ln \left( \frac{10^4}{10^5} \right) = \frac{Ea}{8.314} \left( \frac{1}{1173} - \frac{1}{1023} \right) \]
\[ E = 15.31 \text{ kJ/mol} \]

8. What type of crystal structure is having maximum critical resolved shear stress value?
   (a) BCC   (b) FCC   (c) HCP   (d) SC
   Ans : (a)

9. What is the most favorable slip plane for FCC crystal structure?
   (a) \{110\}   (b) \{112\}   (c) \{111\}   (d) \{123\}
   Ans : (c)

10. The energy associated with surface defects is the maximum for aluminum is due to
    (a) Stacking fault   (b) Twin boundary   (c) Grain boundary   (d) Tilt boundary
    Ans: (c)

11. The stored energy in a copper crystal due to strain field of dislocations is 14 MJ/m\(^3\). The
dislocation density is ................................................. The magnitude of perfect Burgers
    vector of Cu is 0.26 nm. The shear modulus of Cu is 35 GPa.
    (a) 1.18\times10^6 \text{ m}^{-2}   (b) 2.25\times10^6 \text{ m}^{-2}   (c) 3.27\times10^6 \text{ m}^{-2}   (d) 4.19\times10^6 \text{ m}^{-2}
    Ans : G = 35 GPa
    \[ b = 0.26 \text{ nm} \]
    \[ E = 14\times10^6 \text{ J/m}^3 \]
    \[ E = \frac{6b^2}{2} = \frac{35\times10^3 \times (0.26 \times 10^{-9})^2}{2} = 1.18 \times 10^{-9} \text{ J/m} \]
    \[ \rho = \frac{(14\times10^6)}{1.18 \times 10^{-9}} = 1.18 \times 10^6 \text{ m}^{-2} \]

12. In BCC iron a tensile stress of 52 MPa is applied along \([010]\). The critically resolved shear
    stress will be ................................................. MPa, if slip occurs on \((110)\) plane in \([\overline{1} \ 1 \ 1]\)
direction.
    (a) 21.22 MPa   (b) 14.58 MPa   (c) 16.78 MPa   (d) 41.34 MPa
Ans : \( \tau_{cr} = \sigma \cos \phi \cos \lambda \)

\( l_{x} \), \( l_{n} \)

\( x = [010] \), \( n = (110) \), \( d = [\overline{1}11] \)

\[
\cos \lambda = \frac{1}{\sqrt{(1)(3)}} = \frac{1}{\sqrt{3}}, \cos \phi = \frac{1}{\sqrt{(1)(2)}} = \frac{1}{\sqrt{2}}
\]

\( \tau_{cr} = \frac{52}{\sqrt{6}} = 21.22 \text{ MPa} \)

13. A single crystal of copper is loaded under a stress state such that \( \sigma_2 = -\sigma_1 \), \( \sigma_3 = \overline{\tau}_{23} = \overline{\tau}_{31} = \overline{\tau}_{12} = 0 \).

Here \( 1 = [100], 2 = [010], 3 = [001] \).

When the stress \( \sigma_1 = 6 \text{ kPa} \), what is the shear stress, \( \tau \), on the

i. \( (111)[1\overline{0}1] \) slip system?

ii. \( (111)[\overline{1}01] \) slip system?

iii. \( (111)[01\overline{1}] \) slip system?

(a) \( \sqrt{6}, 2\sqrt{6}, -\sqrt{6} \) kPa

(b) \( \sqrt{5}, 3\sqrt{2}, -\sqrt{2} \) kPa

(c) \( \sqrt{7}, 3\sqrt{5}, -\sqrt{5} \) kPa

(d) \( \sqrt{8}, 3\sqrt{7}, -\sqrt{3} \) kPa

Ans: \( \tau = l_{nx}l_{dx}\sigma_{xx} + l_{ny}l_{dy}\sigma_{yy} \)

(i) \( x = [100], n = (111) \)

\[ y = [010], d = [10\overline{1}] \]

\[
\begin{align*}
    l_{nx} &= \frac{1}{\sqrt{(1)(3)}} = \frac{1}{\sqrt{3}} \\
    l_{dx} &= \frac{1}{\sqrt{(1)(2)}} = \frac{1}{\sqrt{2}} \\
    l_{nx} &= \frac{1}{\sqrt{(3)}} = \frac{1}{\sqrt{3}}, l_{dy} = 0
\end{align*}
\]

\[
\tau = \frac{1}{\sqrt{3}} \times \frac{1}{\sqrt{2}} \times 6 \text{ kPa} = \sqrt{6} \text{ kPa}
\]

(ii) \( n = (111), d = [110] \)

\[ x = [100], y = [010] \]

\[
\tau = l_{nx}l_{dx}\sigma_{xx} + l_{ny}l_{dy}\sigma_{yy}
\]
\[ l_{nx} = \frac{1}{\sqrt{3}}, \quad l_{dx} = \frac{1}{\sqrt{2}} \]
\[ l_{ny} = \frac{1}{\sqrt{3}}, \quad l_{dy} = -\frac{1}{\sqrt{2}} \]
\[ \tau = 6 \times \frac{1}{\sqrt{3}} \times \frac{1}{\sqrt{2}} + 6 \times \frac{1}{\sqrt{3}} \times \frac{1}{\sqrt{2}} = 2\sqrt{6}\text{kPa} \]

(iii) \( n = (111), d=[01\overline{1}] \)
\( x = [100], y = [010] \)
\[ l_{nx} = \frac{1}{\sqrt{3}}, \quad l_{dx} = 0 \]
\[ l_{ny} = \frac{1}{\sqrt{3}}, \quad l_{dy} = \frac{1}{\sqrt{2}} \]
\[ \tau = l_{nx}l_{dx}\sigma_{xx} + l_{ny}l_{dy}\sigma_{yy} \]
\[ \tau = 6 \times \frac{1}{\sqrt{3}} \times 0 - 6 \times \frac{1}{\sqrt{3}} \times \frac{1}{\sqrt{2}} = -\sqrt{6}\text{kPa} \]

14. A crystal of aluminium contains \(10^{12}\) m of dislocation per m\(^3\).
(a) Calculate the total amount of energy per m\(^3\) associated with dislocations. Assume that half of the dislocations are edges and half are screws.
(b) What would be the temperature rise if all of this energy could be released as heat?
Data for aluminum: atomic diameter = 0.286 nm, crystal structure = FCC, density = 2.7 Mg/m\(^3\), atomic mass = 27 g/mole, Specific heat = 0.9 J/g°C, Shear modulus G = 70 GPa, Poission’s ratio \( \mu = 0.3 \), \( R = 8.023 \text{ J/mol K} \)

(a) 4.242 kJ/m\(^3\), 1.74×10\(^{-3}\)
(b) 1.844 kJ/m\(^3\), 7.74×10\(^{-3}\)
(c) 11.767 kJ/m\(^3\), 8.34×10\(^{-3}\)
(d) 14.348 kJ/m\(^3\), 10.74×10\(^{-3}\)

\[ W_i^s = \frac{Gb^2}{4\pi} \ln \frac{r}{r_0} \]
\[ = \frac{70 \times 10^9 \times (0.286^{-9})^2}{4\pi} \times \ln \left( \frac{0.5 \times 10^{-6}}{0.286 \times 10^{-9}} \right) \]
\[ = 4.55 \times 10^{-10} \times 7.46 \]
\[ W_i^s = 3.39 \times 10^{-9} \text{J/m} \]
\[ W_i^E = 1.5 \times 3.39 \times 10^{-9} = 5.09 \times 10^{-9} \text{ J/m} \]

\[ E_{\text{total}} = \frac{1}{2} \times \rho \times (W_i^E + W_i^E) = 4.242 \text{ kJ/m}^3 \]

(b) TE = Heat Energy
\[ E_{\text{total}} = \rho \ C_p \Delta T \]
\[ \Rightarrow 4.242 \times 10^3 = 2.7 \times 10^6 \times 0.9 \times \Delta T \]
\[ \Rightarrow \Delta T = 1.74 \times 10^{-3} \text{ C} \]

15. An FCC monocrystal nickel is sheared by \( \gamma = 0.1 \). Assuming that the dislocation density is equal to \( 10^8 \text{ cm}^{-2} \) and it remains constant during deformation. What is the average distance each dislocation will move? If the shear strain rate is \( 10^{-4} \text{ s}^{-1} \), what is the mean velocity of the dislocation? The radius of nickel atom is 0.125 nm.

(a) 0.4 mm, 4.7\times10^{-7} \text{ m/s}
(b) 0.04 mm, 2.25\times10^{-7} \text{ m/s}
(c) 0.17 mm, 5.37\times10^{-8} \text{ m/s}
(d) 1.14 mm, 7.89\times10^{-8} \text{ m/s}

Ans: (a) \( \bar{l} = \frac{\gamma}{\rho b} \)

\[ \rho = 10^8 \text{ cm}^{-2} = 10^{12} \text{ m}^{-2} \]
\[ b = 2r = 0.25 \text{ nm} \]
\[ \gamma = 0.1 \]
\[ \bar{l} = \frac{0.1}{10^{12} \times 0.25 \times 10^{-9}} = 4 \times 10^{-4} \text{ m} = 0.4 \text{ mm} \]

(b) \( \dot{l} = \frac{\dot{\gamma}}{\rho b} \)

\[ \dot{l} = \frac{10^{-4}}{10^{12} \times 0.25 \times 10^{-9}} = 4.7 \times 10^{-7} \text{ m/s} \]