Introduction to crystal elasticity and crystal plasticity

Module 3: Continuum plasticity (Part A)

1. If a cubic cell is subjected to tri-axial hydrostatic tension, the state of shear stress is
   Ans. Zero in all the planes

2. In small strain tensor, the mathematical shear strain \((\epsilon_{ij})\) is .......... of engineering shear strain.
   Ans. Half \((1/2)\)

3. __________ quantity is a physical quantity which has no specific direction but different values in different direction.
   Ans. Tensor

4. The extremities of any diameter on Mohr’s circle represent.
   a. Principal stress  
   b. Normal stress on plane at 45 degree  
   c. Shear stress on plane 45 degree  
   d. None of the above
   Ans: a

5. Hooke’s law is valid for which of the following region in stress-strain curve of metals?
   (a) Elastic region (b) Plastic region (c) Both elastic and plastic region (d) none of the above
   Ans: (a)

6. Find the principal stresses for the stress tensor given below.

\[
\begin{bmatrix}
3 & -1 & 0 \\
-1 & 3 & 0 \\
0 & 0 & 1 \\
\end{bmatrix}
\]

Ans: \[\text{Characteristics equation } |\sigma - \lambda I| = 0\]

\[
\begin{vmatrix}
3 - \lambda & -1 & 0 \\
-1 & 3 - \lambda & 0 \\
0 & 0 & 1 - \lambda \\
\end{vmatrix} = 0
\]

\[
(3 - \lambda)((3 - \lambda)(1 - \lambda)) + (-1 + \lambda) = 0
\]

\[
\Rightarrow -\lambda^3 + 7\lambda^2 - 14\lambda + 8 = 0
\]

\[
\Rightarrow \lambda^3 - 7\lambda^2 + 14\lambda - 8 = 0
\]
The principal stresses are

\[ \lambda_1 = 1; \lambda_2 = 2; \text{ and } \lambda_3 = 4 \]

7. The stress state at a point is described by \[ \begin{bmatrix} 6 & 2 & 2 \\ 2 & 0 & 4 \\ 2 & 4 & 0 \end{bmatrix} \]. Find the normal and shear stresses upon a plane whose normal is defined by the direction cosines \( l = m = n = 1/\sqrt{3} \). All stress components are in MPa.

Ans: \[ [\sigma] = \begin{bmatrix} 6 & 2 & 2 \\ 2 & 0 & 4 \\ 2 & 4 & 0 \end{bmatrix} = 2 \begin{bmatrix} 3 & 1 & 1 \\ 1 & 0 & 2 \\ 1 & 2 & 0 \end{bmatrix} \]

\[ \therefore \text{Characteristics equation } |\sigma - \lambda I| = 0 \]

\[ \begin{vmatrix} 3 - \lambda & 1 & 1 \\ 1 & -\lambda & 2 \\ 1 & 2 & -\lambda \end{vmatrix} = 0 \]

\[ \Rightarrow (3 - \lambda)(\lambda^2 - 4) - (\lambda - 3) + (2 + \lambda) = 0 \]

\[ \Rightarrow -\lambda^3 + 3\lambda^2 - 12 + 4\lambda + \lambda + 2 + 2 + \lambda = 0 \]

\[ \Rightarrow -\lambda^3 + 3\lambda^2 + 6\lambda - 8 = 0 \]

\[ \Rightarrow \lambda^3 - 3\lambda^2 - 6\lambda + 8 = 0 \]

\[ \therefore \lambda_1 = 1; \lambda_2 = -2; \lambda_3 = 4 \]

\[ \sigma_1 = 2 \times 4 = 8; \sigma_2 = 2 \times 1 = 2; \sigma_3 = -2 \times 2 = -4 \]

Direction cosines indicate the octahedral plane.

\[ \therefore \sigma_p = \frac{1}{3}(\sigma_{11} + \sigma_{22} + \sigma_{33}) = \frac{1}{3}(6 + 0 + 0) = 2 \text{ Mpa} \]

\[ \therefore \tau_{oct} = \frac{1}{3}((\sigma 1 - \sigma 2)^2 + (\sigma 2 - \sigma 3)^2 + (\sigma 3 - \sigma 1)^2) \]

\[ = \frac{1}{3}(6^2 + 6^2 + 12^2) = \frac{1}{3}\sqrt{216} = 2\sqrt{6} = 4.9 \text{ Mpa} \]

8. Find the deviatoric components of \( \sigma \) given below and find the principal stresses. \( \sigma = \begin{bmatrix} 6 & 2 & 2 \\ 2 & 0 & 4 \\ 2 & 4 & 0 \end{bmatrix} \). (Note: follow the approach of finding principal stresses of deviatoric components)

Ans: \[ \therefore \text{Characteristics equation } |\sigma - \lambda I| = 0 \]

\[ \begin{vmatrix} 1 - \lambda & 2 & 3 \\ 2 & 2 - \lambda & 4 \\ 3 & 4 & 3 - \lambda \end{vmatrix} = 0 \]

\[ \Rightarrow (1 - \lambda)(2 - \lambda)(3 - \lambda) - 16) - 2((6 - 2\lambda) - 12) + 3(8 - (6 - 3\lambda)) = 0 \]

\[ \Rightarrow \lambda^3 - 6\lambda^2 - 18\lambda - 8 = 0 \]

\[ \therefore \lambda_1 = \sigma_1 = 8.288; \lambda_2 = \sigma_2 = -0.557; \lambda_3 = \sigma_3 = -1.73 \]

Hydrostatic stress, \( \sigma_h = \frac{\sigma_1 + \sigma_2 + \sigma_3}{3} = 2 \)

We know, \( \sigma_1 = \sigma_1^1 + \sigma_h \)
Deviatoric components, \( \sigma_1 \) = \( \sigma_1 - \sigma_n \) = 8.288 – 2 = 6.288 MPa
\( \sigma_2 = \sigma_2 - \sigma_n \) = -2.557
\( \sigma_3 = \sigma_1 - \sigma_n \) = -3.73

9. A cylindrical specimen of steel having an original diameter of 10 mm is tensile tested to fracture and found to have engineering fracture strength of 460 MPa. The yield strength, elastic modulus and ultimate tensile strength of material are 570 MPa, 420 MPa and 620 MPa, respectively. If its cross-sectional diameter at fracture is 9 mm, the true fracture stress is …………………….. MPa.

Ans:
Given engineering fracture strength \( \sigma^E = 460 \) MPa

Engineering strain \( \varepsilon^E = \frac{d_0 - d_i}{d_0} = \frac{10-9}{10} = 0.1 \)

True fracture strength \( \sigma^F = \sigma^E (1 + \varepsilon^E) = 460 (1+0.1) = 506 \) MPa

10. The strain hardening exponent “n” for an alloy in which a true stress of 415 MPa produces a true strain of 0.10 is …………………….. Assume a value of 1035 MPa for the constant “K”.

Ans: From the Hollomon equation or Power law \( \sigma = K \varepsilon^n \)

\( \varepsilon^n = \frac{\sigma}{K} = \frac{415}{1035} = 0.4 \)

\( \therefore \) Strain hardening exponent \( n = \ln \frac{0.4}{0.1} = 0.39 \)

11. According to engineering stress-strain diagram the ultimate tensile strength of a material is 65 GPa corresponding to the strain of 0.2. What is the value of true stress?

Ans : True stress \( \sigma^T = \sigma^E (1 + \varepsilon) = 65 (1+0.2) = 78 \) GPa

12. In an uniaxial deformation, the material is loaded beyond yield point and reaches at the stress point of 800 MPa and strain point of 0.2. The Young’s modulus of this material is 80 GPa. If the load is removed, what will be the elastic recovery?

Ans: Given data
\( \sigma = 800 \) MPa
\( \varepsilon = 0.2 \)
\( E = 80 \) GPa

Now the material is loaded beyond the yield point. Once the material is loaded beyond the yield point only the elastic part is recoverable. Plastic deformation part is not recoverable.

Plastic strain \( \varepsilon_p = 800\text{MPa}/80\text{GPa} \)
\[ ε_p = 0.01 \]

Now the elastic recovery is \( δ = ε - ε_p = 0.2 - 0.01 = 0.19 \)

13. Suppose a unit cell of BCC crystal is subjected to uniaxial stress of 60 MPa along [100] direction. What will be the maximum shear stress on (011) and (101) planes respectively?

Ans: 0 and 60/2 i.e. 0 and 30 MPa

\[ \tau = l_{nx}l_{dx}σ_x \]

Refer: class notes

14. Consider an aluminum single crystal that has been stretched in tension applied parallel to \( x = [100] \) by 250 kPa, in compression parallel to \( y = [010] \) by 50 kPa and tension with 10 kPa by \( z = [001] \). Assume that slip occurred on the (111) in the [110] direction and only on the slip system.

What is the shear stress on slip plane?

Ans: Shear stress, \( τ_{nd} = l_{nx}l_{dx}σ_{xx} + l_{ny}l_{dy}σ_{yy} + l_{nz}l_{dz}σ_{zz} \)

\( n = [111], d = [1\overline{1}0] \)

\( x = [100], y = [010], z = [001] \)

\[ l_{nx} = \frac{1}{\sqrt{3} \times \sqrt{1}} = \frac{1}{\sqrt{3}}, l_{dx} = \frac{1}{\sqrt{2} \times \sqrt{1}} = \frac{1}{\sqrt{2}} \]

\[ l_{ny} = \frac{1}{\sqrt{3} \times \sqrt{1}} = \frac{1}{\sqrt{3}}, l_{dy} = \frac{-1}{\sqrt{2} \times \sqrt{1}} = -\frac{1}{\sqrt{2}} \]

\[ l_{nz} = \frac{1}{\sqrt{3} \times \sqrt{1}} = \frac{1}{\sqrt{3}}, l_{dz} = \frac{0}{\sqrt{2} \times \sqrt{1}} = 0 \]

\[ ∴ τ_{nd} = \frac{1}{\sqrt{3}} \times \frac{1}{\sqrt{2}} \times 250 + \frac{1}{\sqrt{3}} \times \left(-\frac{1}{\sqrt{2}}\right) \times 50 + 0 = 81.65 \text{ kPa} \]