1. For an isotropic material, what is/are the number(s) of independent elastic constants?

(a) 1  (b) 2  (c) 3  (d) 4  

Marks 1

Ans (b)

2. For isotropic material, the directional elastic modulus is equal to:

(a) \( \frac{1}{S_{12} S_{11}} \)  (b) \( \frac{1}{S_{12}} \)  (c) \( \frac{1}{S_{44}} \)  (d) \( \frac{1}{S_{11}} \)

Marks 1

Ans (d)

3. If the elastic modulus is different along three perpendicular directions, the material is said to be

(a) isotropic  (b) anisotropic  (c) orthotropic  (d) monoclinic

Ans (c)

4. A brittle material has

(a) No elastic zone  (b) No plastic zone  (c) Large plastic zone  (d) None of these

Ans (b)

5. In a composite body, consisting of two different materials …….. will be same in both materials along longitudinal direction.

(a) Stress  (b) Strain  (c) Both stress and strain  (d) None of these

Ans (b)

6. A 200 × 100 × 50 mm steel block is subjected to a hydrostatic pressure of 10MPa. The Young’s modulus and Poisson’s ratio of the material are 200 GPa and 0.3 respectively. The change in the volume of the block in \( \text{mm}^3 \) is

(a) 30  (b) 60  (c) 90  (d) 100

Ans (b)

**Solution**

For hydrostatic pressure \( \sigma_x = \sigma_y = \sigma_z = \sigma \)

Longitudinal strain \( \varepsilon_x = \varepsilon_y = \varepsilon_z = \varepsilon = \frac{\sigma}{E} - \mu \frac{\sigma}{E} - \mu \frac{\sigma}{E} \)
\[ \varepsilon = \frac{\sigma}{E} (1-2\mu) \]

Volumetric strain \( \varepsilon_v = \frac{\delta V}{V} = 3\varepsilon = \frac{3\sigma}{E} (1-2\mu) \)

\[ \therefore \delta V = \frac{3\times10^{-3}}{200\times10^3} (1-2 \times 0.3 \{ 200 \times 100 \times 50 \} \]

\[ = 60 \text{ mm}^3 \]

7. Calculate Young’s modulus for an iron crystal when tension is applied along \(<122>\) direction.

Hint: Directly estimate \( \left( \frac{1}{E_d} \right) \)

\[ (a) \ 4.24 \ (\text{TPa})^{-1} \quad (b) \ 4.77 \ (\text{TPa})^{-1} \quad (c) \ 5.73 \ (\text{TPa})^{-1} \quad (d) \ 3.977 \ (\text{TPa})^{-1} \]

**Ans** \( (d) \)

**Solution**

\[ \frac{1}{E_d} = S_{11} + (-2S_{11} + 2S_{12} + S_{44})(\beta^2\gamma^2 + \gamma^2\alpha^2 + \alpha^2\beta^2) \]

For the direction family \(<122>\)

\[ \alpha = \frac{1}{3}, \beta = \frac{2}{3}, \gamma = \frac{2}{3} \]

So \[ \frac{1}{E_d} = S_{11} + (-2S_{11} + 2S_{12} + S_{44}) \left( \frac{16}{81} + \frac{4}{81} + \frac{4}{81} \right) \]

\[ = S_{11} + (-2S_{11} + 2S_{12} + S_{44}) \left( \frac{8}{27} \right) \]

\( S_{11} = 7.56 \ (\text{TPa})^{-1}, \ S_{12} = -2.78 \ (\text{TPa})^{-1}, \ S_{44} = 8.59 \ (\text{TPa})^{-1} \)

\[ \frac{1}{E_d} = 7.56 + (-2 \times 7.56 + 2 \times (-2.78) + 8.59) \times \left( \frac{8}{27} \right) \]

\[ \frac{1}{E_d} = 3.977 \ (\text{TPa})^{-1} \]

8. Consider a thin-walled tube, capped at each end and loaded under internal pressure. Calculate the ratio of the axial strain to the hoop strain, assuming that the deformation is elastic. Assume Poisson’s ratio \( \mu = 1/3. \)

\[ (a) \ 0.4 \quad (b) \ 0.35 \quad (c) \ 0.3 \quad (d) \ 0.2 \]

**Ans** \( (d) \)

**Solution**

We know, Hoop stress \( \sigma_h = \frac{PD}{2t} \)
And axial stress \((\sigma_a) = \frac{PD}{4t}\)

\[
\varepsilon_a = \frac{1}{E} (\sigma_a - \mu\sigma_h) \quad \text{and} \quad \varepsilon_h = \frac{1}{E} (\sigma_h - \mu\sigma_a)
\]

\[
\frac{\varepsilon_a}{\varepsilon_h} = \frac{\sigma_a - \nu\sigma_h}{\sigma_h - \nu\sigma_a} = \frac{\frac{PD}{4t} - \frac{1}{2} \frac{PD}{t^2} \frac{1}{3}}{\frac{PD}{2t} \frac{1}{1-\nu}} = \frac{\frac{3-2}{6}}{\frac{1}{5}} = \frac{1}{5}
\]

\[
\therefore \frac{\varepsilon_a}{\varepsilon_h} = \frac{1}{5}
\]

9. Pressure, \(p\) affects the \(c/a\) ratio of hexagonal crystals such that \(\sigma_1 = \sigma_2 = \sigma_3 = -p\). The correct expression for \(d(c/a)/dp\) in terms of the elastic constants and \(c/a\) is

(a) \(- (c/a)(s_{13} + s_{33} - s_{11} - s_{12})\)

(b) \((c/a)(s_{13} + s_{33} - s_{11} - s_{12})\)

(c) \(- (c/a)(s_{13} + s_{33} - s_{11})\)

(d) \(- (c/a)(s_{13} + s_{33} - s_{11} + s_{12})\)

Ans (a)

Solution

\(d(c/a) = (ad - cda)/a^2 = (c/a)(dc/c - da/a) = (c/a)(de_3 - de_1).\)

Now substituting

\(de_1 = s_{11} d\sigma_1 + s_{12} d\sigma_2 + s_{13} d\sigma_3 = -(s_{11} + s_{12} + s_{13}) dp\)

and

\(de_3 = s_{13} d\sigma_1 + s_{13} d\sigma_2 + s_{33} d\sigma_3 = -(2s_{13} + s_{33}) dp,\)

\(d(c/a)/dp = -(c/a)(2s_{13} + s_{33} - s_{11} - s_{12} - s_{13})\)

\(= -(c/a)(s_{13} + s_{33} - s_{11} - s_{12}).\)

Hexagonal cell showing the \(c\)-axis and a direction \(d\). The dimensions of the cell area and \(d\).
10. A cubic crystal is loaded with a tensile stress of 3 MPa applied along the [110] direction. Find the shear stress on the (111) plane in the [101] direction. 

**Marks 2**

(a) 1.22 MPa  (b) 1.37 MPa  (c) 0.75 MPa  (d) 0.5 MPa

**Ans (a)**

**Solution**

In a cubic crystal, the normal to a plane has the same indices as the plane, so the normal to (111) is [111]. Also, in a cubic crystal, the cosine of the angle between two directions is given by the dot product of unit vectors in those directions.

For example, the cosine of the angle between \([u_1v_1w_1]\) and \([u_2v_2w_2]\) is equal to

\[
\frac{u_1 u_2 + v_1 v_2 + w_1 w_2}{\sqrt{(u_1^2 + v_1^2 + w_1^2)(u_2^2 + v_2^2 + w_2^2)}}
\]

Designating [110] as x, [101] as d and [111] as n

Shear stress can be defined as

\[
\tau_{nd} = l_{nx} l_{dx} \sigma_{xx}
\]

\[
\tau_{nd} = \frac{1 \times 1 + 1 \times 1 + 0 \times 1}{\sqrt{(1^2 + 1^2 + 0^2)(1^2 + 1^2 + 1^2)}} \times \frac{1 \times 1 + 1 \times 0 + 0 \times 1}{\sqrt{(1^2 + 1^2 + 0^2)(1^2 + 0^2 + 1^2)}} \times 3 \text{ MPa}
\]

\[
\tau_{nd} = 1.22 \text{ MPa}
\]

11. Zinc has the following elastic constants. 

**Marks 2**

\[s_{11} = 0.84 \times 10^{-11} \text{ Pa}^{-1} ; s_{33} = 2.87 \times 10^{-11} \text{ Pa}^{-1} ; s_{12} = 0.11 \times 10^{-11} \text{ Pa}^{-1}\]

\[s_{13} = -0.78 \times 10^{-11} \text{ Pa}^{-1} ; s_{44} = 2.64 \times 10^{-11} \text{ Pa}^{-1} ; s_{66} = 2(s_{11} - s_{12})\]

The bulk modulus of zinc is

(a) 31.4 GPa  (b) 36 GPa  (c) 3.14 GPa  (d) 53.6 GPa

**Ans (a)**

**Solution**

We have,

\[
E = \frac{1}{S_{11}} = \frac{1}{0.84 \times 10^{-11} \text{ Pa}} = 1.19 \times 10^{11} \text{ Pa}
\]
Again, 

\[ E = 3B(1 - 2\nu) \]

Also, 

\[ \nu = -\frac{S_{12}}{S_{11}} = -\frac{0.11 \times 10^{-11}}{0.84 \times 10^{-11}} = -0.13 \]

\[ \therefore B = 31.4 \text{ GPa} \]

12. A steel bar is loaded axially with a load magnitude of \( P = 100 \text{ N} \) as shown in figure below. What is the strain energy stored in the bar. Assume \( E = 210 \text{ GPa} \).

![Steel bar diagram](image)

(a) 2.38X10\(^{-4}\) N/m  
(b) 4.76X10\(^{-4}\) N/m  
(c) 2.38X10\(^{-6}\) N/m  
(d) 5.72X10\(^{-4}\) N/m  

Ans (a)

Solution

Given data,

Load applied \( P = 100 \text{ N} \)  
Length of the bar \( L = 1000 \text{ mm} \)  
Young’s modulus \( E = 210 \text{ GPa} \)

From the figure the cross sectional area over which load is applied \( A = 100 \text{ mm} \times 100 \text{ mm} = 0.01 \text{ m}^2 \)

The strain energy stored in the bar \( U_{\text{total}} = \frac{P^2L}{2EA^2} = \frac{100^2 \times 10^{-6}}{2 \times 210 \times 10^9 \times 0.01^2} \text{ N/m} = 2.38 \times 10^{-4} \text{ N/m} \)

13. A hydrostatic compressive stress applied to a material with cubic symmetry results in a dilation of \(-3 \times 10^{-5}\). The three independent elastic constants of the material are \( C_{11} = 50 \text{ GPa} \), \( C_{12} = 40 \text{ GPa} \), and \( C_{13} = 40 \text{ GPa} \). Compute the applied hydrostatic stress.

(a) -1.3 MPa  
(b) -13 MPa  
(c) -433 kPa  
(d) 0.13 MPa  

Ans (a)
Solution

Dilation is the sum of the principal strain components:

\[ \varepsilon = \varepsilon_1 + \varepsilon_2 + \varepsilon_3 = -3 \times 10^{-5} \]

Cubic symmetry implies that

\[ \varepsilon_1 = \varepsilon_2 = \varepsilon_3 = -1 \times 10^{-5} \]

And

\[ \varepsilon_4 = \varepsilon_5 = \varepsilon_6 = 0 \]

From Hooke’s law

\[ \sigma_i = C_{ij} \varepsilon_j \]

And

\[ \sigma_1 = C_{11} \varepsilon_1 + C_{12} \varepsilon_2 + C_{13} \varepsilon_3 \]

\[ \therefore \sigma_p = \sigma_1 = (50 + 40 + 40)(-1 \times 10^{-5}) \text{GPa} = -1.3 \text{MPa} \]

14. The values \( E \) and \( \nu \) for niobium are given as 150GPa and 0.34. Compute shear modulus.  
(Marks 2)

(a) 55.97 GPa  (b) 5.59 GPa  (c) 111.94 GPa  (d) 113.63 GPa

Ans (a)

Solution

Given \( E = 150 \text{ GPa} \) and \( \nu = 0.34 \)

From the relation \( E = 2G(1+\nu) \)

\[ G = 55.97 \text{ GPa} \]

15. In a circuit box, a copper rod at room temperature of 25\(^0\)C has a gap of 0.3 mm between the end of the rod Q and the rigid wall. If temperature of the rod increases to 70\(^\circ\)C, then what is the compressive stress acting on the rod PQ? (Assume Young’s modulus = 100 Gpa & \( \alpha = 15 \times 10^{-6} /\circ\)C)
a. 6 Mpa. 7.5 Mpa. 8.2 Mpa. None of the above

Ans : (b)

Solution:

Given: Gap = 0.3 mm, Young's modulus = 100 Gpa & α = 15 x 10^{-6} /°C,

Free expansion of copper rod = α t L = 15 x 10^{-6} x (70 - 25) x 500 = 0.3375

Actual expansion prevented = Free expansion of copper rod - gap
= 0.3375 - 0.3 = 0.0375

Temperature strain (e) = (δL/L) = (0.0375/ 500) = 7.5 x 10^{-5}

Temperature stress = e E = (7.5 x 10^{-5} x 100 x 10^{3}) = 7.5 Mpa

Compressive stress acting on the rod PQ = 7.5 Mpa