Modeling and Simulation of Discrete Event Systems
Solution of Assignment 6

Q1. Failure to recognize and deal with randomness in simulation output can lead to

(a) Serious errors
(b) Misinterpretation
(c) Bad decision
(d) All of above

Q2. The type of simulation for which there is no natural event E to specify length of each run is known as

(a) Terminating
(b) **Non terminating**
(c) Steady state
(d) None of above

Refer the following for Q3 and Q7.

Results of 10 independent replications of a bank model are presented as follows:

Replication	Number served	Finish time (hours)	Average delay in queue (minutes)	Average queue length
1	550	4.9	1.81	2.5
2	650	3.8	1.80	3.5
3	580	5.6	2.30	2.9
4	495	4.3	1.67	3.4
5	634	6.8	2.34	4.1
6	567	6.1	1.74	4.2
7	589	5.3	2.15	3.5
8	543	7.1	2.17	3.8
9	592	6.9	1.73	2.9
10	687	4.1	1.55	3.6

Q3. The approximate 90% confidence interval for the expected average queue length will be

(a) [3.44 ± 0.72]
(b) **[3.44± 0.31]**
(c) [3.88 ± 0.42]
(d) [3.88 ± 0.31]

Q4. The approximate 95% confidence interval for the expected average delay in queue(in minutes) will be
(a) **[1.92 ± 0.20]**
(b) [1.89 ± 0.20]
(c) [1.92 ± 0.38]
(d) [1.89 ± 0.17]

Q5. The approximate 95% confidence interval for the expected number of persons served will be
(a) [577.8 ± 60.58]
(b) [588.7 ± 60.58]
(c) **[588.7 ± 40.17]**
(d) [577.8 ± 40.17]

Q6. The approximate 97.5% confidence interval for the expected finish time (in hours) will be
(a) **[5.49 ± 1.03]**
(b) [5.49 ± 0.28]
(c) [7.49 ± 0.33]
(d) [7.49 ± 0.17]

Q7. The approximate 97.5% confidence interval for the expected average delay in queue(in minutes) will be
(a) [1.92 ± 0.62]
(b) [1.92 ± 0.88]
(c) **[1.92 ± 0.24]**
(d) [1.89 ± 0.17]

Refer the following for Q8 to Q10.

Average total cost for two policies, X_1 and X_2 are to be compared, whose values are given for 5 independent replications. For knowing whether μ1 differs from μ2, confidence interval for the difference ζ = μ1 - μ2 is to be created using paired t-approach.

Replication	X_1	X_2
1	127	118
2	124	120
3	127	122
4	123	123
5	127	119

Q8. The approximate 90% confidence interval for ζ will be
(a) [5.2 ± 5.39]
(b) [4.9 ± 3.78]
(c) [4.9 ± 5.67]

(d) **[5.2 ± 3.39]**

Q9. The approximate 98% confidence interval for ζ will be
(a) [5.2 ± 3.28]
(b) **[5.2 ± 5.97]**
(c) [4.9 ± 4.39]
(d) [4.9 ± 6.74]

Q10. The approximate 95% confidence interval for ζ will be
(a) [5.2 ± 5.39]
(b) [4.9 ± 3.78]
(c) [4.9 ± 5.67]
(d) **[5.2 ± 4.43]**

Refer the following for Q11 to Q15.

For three different inventory systems, average total cost per month for five independent replications of each of the system are given below. Considering first system as the standard one, using paired-t approach construct two confidence intervals for the difference in the mean (i.e. for μ_2- μ_1 and μ_3- μ_1),

Replication	**X_1**	**X_2**	**X_3**
1	127	118	121
2	124	120	129
3	127	122	121
4	123	123	124
5	127	119	127

Q11. The confidence interval for μ_2- μ_1 with overall confidence level of 90% will be
(a) **[-5.2 ± 4.43]**
(b) [-5.2 ± 6.21]
(c) [-5.7 ± 5.98]
(d) [5.2 ± 3.75]

Q12. The confidence interval for μ_3- μ_1 with overall confidence level of 98% will be
(a) [1.7 ± 1.21]
(b) **[-1.2 ± 9.80]**
(c) [-1.2 ± 7.11]
(d) [-1.2 ± 8.58]

Q13. The confidence interval for μ_2- μ_1 with overall confidence level of 98% will be
(a) **[-5.2 ± 7.33]**
(b) [-5.2 ± 6.21]
(c) [-5.7 ± 5.98]
(d) [5.2 ± 3.75]

Q14. The confidence interval for μ_3- μ_1 with overall confidence level of 90% will be

(a) [1.7 ± 1.21]
(b) [-1.2 ± 0.29]
(c) [-1.2 ± 7.11]
(d) **[-1.2 ± 5.92]**

Q15. Which of the following statement is correct among the following?
(a) **System 2 differs from standard at overall confidence level of 90%.**
(b) System 3 differs from standard at overall confidence level of 90%.
(c) System 2 differs from standard at overall confidence level of 98%.
(d) System 3 differs from standard at overall confidence level of 98%.

Percentage points of the Student's t distribution with υ degrees of freedom

Approximate percentage of confidence											
υ	***90***	***95***	***97.5***	***99***	***99.5***	***99.75***	***99.9***	***99.95***	***99.975***	***99.99***	***99.995***
1	3.08	6.31	12.71	31.82	63.66	127.32	318.31	636.62	1273.24	3183.10	6366.20
2	1.89	2.92	4.30	6.96	9.22	14.09	22.33	31.60	44.70	70.70	99.99
3	1.64	2.35	3.18	4.54	5.84	7.45	10.21	12.92	16.33	22.20	28.00
4	1.53	2.13	2.78	3.75	4.60	5.60	7.17	8.61	10.31	13.03	15.54
5	1.48	2.02	2.57	3.37	4.03	4.77	5.89	6.87	7.98	9.68	11.18
6	1.44	1.94	2.45	3.14	3.71	4.32	5.21	5.96	6.79	8.02	9.08
7	1.42	1.90	2.37	3.00	3.50	4.03	4.79	5.41	6.08	7.06	7.88
8	1.40	1.86	2.31	2.90	3.36	3.83	4.50	5.04	5.62	6.44	7.12
9	1.38	1.83	2.26	2.82	3.25	3.69	4.30	4.78	5.29	6.01	6.59
10	1.37	1.81	2.23	2.76	3.17	3.58	4.14	4.59	5.05	5.69	6.21
11	1.36	1.80	2.20	2.72	3.11	3.50	4.03	4.44	4.86	5.45	5.92
12	1.36	1.78	2.18	2.68	3.06	3.43	3.93	4.32	4.72	5.26	5.69
13	1.35	1.77	2.16	2.65	3.01	3.37	3.85	4.22	4.60	5.11	5.51
14	1.35	1.76	2.15	2.63	2.98	3.33	3.79	4.14	4.50	4.99	5.36
15	1.34	1.75	2.13	2.60	2.95	3.29	3.73	4.07	4.42	4.88	5.24
16	1.34	1.75	2.12	2.58	2.92	3.25	3.69	4.02	4.35	4.79	5.13
17	1.33	1.74	2.11	2.57	2.90	3.22	3.65	3.97	4.29	4.71	5.04
18	1.33	1.73	2.10	2.55	2.88	3.20	3.61	3.92	4.23	4.65	4.97
19	1.33	1.73	2.09	2.54	2.86	3.17	3.58	3.88	4.19	4.59	4.90
20	1.33	1.73	2.09	2.53	2.85	3.15	3.55	3.85	4.15	4.54	4.84
30	1.31	1.70	2.04	2.46	2.75	3.03	3.39	3.65	3.90	4.23	4.48
40	1.30	1.68	2.02	2.42	2.70	2.97	3.31	3.55	3.79	4.09	4.32
50	1.30	1.68	2.01	2.40	2.68	2.94	3.26	3.50	3.72	4.01	4.23
60	1.30	1.67	2.00	2.39	2.66	2.91	3.23	3.46	3.68	3.96	4.17
100	1.29	1.66	1.98	2.36	2.63	2.87	3.17	3.39	3.60	3.86	4.05
200	1.29	1.65	1.97	2.35	2.60	2.84	3.13	3.34	3.54	3.79	3.97
500	1.28	1.65	1.97	2.33	2.59	2.82	3.11	3.31	3.50	3.75	3.92
∞	1.28	1.65	1.96	2.33	2.58	2.81	3.10	3.30	3.49	3.73	3.91

Solutions for Q 3 to Q7:

	Number served	Finish time (hours)	Average delay in queue (minutes)	Average queue length
B	**C**	**D**	**E**	**F**
1	550	4.9	1.81	2.5
2	650	3.8	1.80	3.5
3	580	5.6	2.30	2.9
4	495	4.3	1.67	3.4
5	634	6.8	2.34	4.1
6	567	6.1	1.74	4.2
7	589	5.3	2.15	3.5
8	543	7.1	2.17	3.8
9	592	6.9	1.73	2.9
10	687	4.1	1.55	3.6
Average	**588.7**	**5.49**	**1.925144**	**3.44**
Sample Var. $S^2(10)$	3155.122	1.474333	0.08068	0.293778
H.L.(90%)	$t_{9,0.95}$* sqrt(sample variance/10)= 1.833 *sqrt (sample variance/10)			
H.L.(90%)	32.55896	0.702666	0.164374	**0.313661** **(Ans 3)**
H.L.(95%)	$t_{9,0.975}$* sqrt(sample variance/10)= 2.26 *sqrt (sample variance/10)			
H.L.(95%)	**40.17915** **(Ans 5)**	0.868541	**0.203177** **(Ans 4)**	0.387706
H.L.(97.5%)	$t_{9,0.9875}$* sqrt(sample variance/10)= 2.685 *sqrt (sample variance/10)			
H.L.(97.5%)	47.69275	**1.030961** **(Ans 6)**	**0.241172** **(Ans 7)**	0.460208

(Method of calculation of half length and confidence interval)

Calculations are done in following manner (for example, for average queue length):

- *Average* ($F_{average}$)= Sum (F_1 to F_{10})/10 = 3.44
- *Sample variance*= Sum ((F_i-$F_{average}$)/9) (For i from 1 to 10)
- *Half length* = $t_{n-1,1-\alpha/2}$* sqrt(sample variance/n)   (n is 10 in this example)

(For 90% confidence level, 1-α =0.9 i.e. α = 0.1, so 1-α/2=0.95 and in the similar way, $t_{n-1,1-\alpha/2}$ can be calculated for different confidence level values.

- *Confidence interval* = average ± half length

Solutions for Q 8 to Q10:

j	**X_{1j}**	**X_{2j}**	**$Z_{j=Xij-X2j}$**
1	127	118	9
2	124	120	4
3	127	122	5
4	123	123	0
5	127	119	8
Average	125.6	120.4	**5.2**
Sam. Var.	3.8	4.3	12.7
Half length 90%	$t_{4,0.95}$* sqrt(sample variance/5)= 2.13 *sqrt (sample variance/5)		
Half length 90%			**3.394661** **(Ans 8)**
Half length 95%	$t_{4,0.975}$* sqrt(sample variance/5)= 2.78 *sqrt (sample variance/5)		
Half length 95%			**4.430591** **(Ans 10)**
Half length 98%	$t_{4,0.99}$* sqrt(sample variance/5)= 3.75 *sqrt (sample variance/5)		
Half length 98%			5.976517 **(Ans 9)**

- *Confidence interval* = average ± half length

Solutions for Q 11 to Q14:

j	X_{1j}	X_{2j}	$X3_j$	$Z_{j=X2j-X1j}$	$Z_{j=X3j-X1j}$
1	127	118	121	-9	-6
2	124	120	129	-4	5
3	127	122	121	-5	-6
4	123	123	124	0	1
5	127	119	127	-8	0
Average	125.6	120.4	124.4	**-5.2**	**-1.2**
Sam. Var.	3.8	4.3	12.8	12.7	22.7
Half length 90% (overall confidence)	$t_{4,0.975}$* sqrt(sample variance/5)= 2.78 *sqrt (sample variance/5)				
Half length 90%				**4.430591 (Ans 11)**	**5.923423 (Ans 14)**
Half length 98% (Overall confidence)	$t_{4,0.995}$* sqrt(sample variance/5)= 4.60 *sqrt (sample variance/5)				
Half length 98%				**7.331194 (Ans 13)**	**9.801347 (Ans 12)**

- *Confidence interval* = average ± half length

(For 90% overall confidence level, individual confidence level is to be maintained at 95%, hence 1-α/2 becomes equal to 0.975 and accordingly half-length is computed).

Answer to Q15:

90% Confidence interval for μ_2- μ_1: -5.2 ± 4.43 (does not contain 0 in the interval, hence differ from standard)

98% Confidence interval for μ_2- μ_1: -5.2 ± 7.33 (does contain 0 in the interval, hence do not differ from standard)

90% Confidence interval for μ_3- μ_1: -1.2 ± 5.92 (does contain 0 in the interval, hence do not differ from standard)

98% Confidence interval for μ_3- μ_1: -1.2 ± 9.80 (does contain 0 in the interval, hence do not differ from standard)