Unit 8 - Week 6

Course outline

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Pre-Requisite Assignment

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Week 6

- Westergaard Solution of Stress Field for Mode-I
- Displacement Field for Mode-I
- Relationship between KI and GI
- Feedback for week 6
- Quiz : Assignment 6

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Assignment 6

The due date for submitting this assignment has passed.
As per our records you have not submitted this assignment.

Due on 2019-09-11, 23:59 IST.

Instructions for answering numerical questions

1. In all numerical type questions, you are expected to round off the answers to two decimal places accuracy unless otherwise specified.
   Examples: 1. Ans: 9.825, you report as 9.83
   2. Ans: 9.8, you report as 9.80
   3. Ans: 9, you report as 9.00

This style of reporting is essential for computer based automated correction of your answers.

2. The answers for various quantities asked are to be reported in the following units unless otherwise specified,
Stress- MPa, Stress Intensity Factor- MPa√m, Strain energy- Nmm, Energy release rate- J/m², deflection – mm,

1) For Mode-I loading, stress function Z, with crack center as origin, suggested by Westergaard is *1 point*

- $Z = \dfrac{\sigma}{\sqrt{z^2 - a^2}}$
- $Z = \dfrac{\sigma z}{\sqrt{a^2 - z^2}}$
- $Z = \dfrac{\sigma z}{\sqrt{z^2 - a^2}}$
- $Z = \dfrac{\sigma(z-a)}{\sqrt{z^2 - a^2}}$

No, the answer is incorrect.
Score: 0

Accepted Answers:
$Z = \dfrac{\sigma z}{\sqrt{z^2 - a^2}}$

2) Stress intensity factor has a unit of *1 point*

- $\dfrac{Force}{Area}\sqrt{length}$
- $Force\sqrt{length}$
- $\dfrac{Force}{Area}$
- $\dfrac{Area}{Force}\sqrt{length}$

No, the answer is incorrect.
Score: 0

Accepted Answers:
$\dfrac{Force}{Area}\sqrt{length}$

3) The energy release rate for Mode-I crack is given by *1 point*

- $\dfrac{\sigma^2 a}{E}$
- $\dfrac{\pi\sigma^2 a}{G}$
- $\dfrac{\pi\sigma^2 E}{a}$
- $\dfrac{\pi\sigma^2 a}{E}$

No, the answer is incorrect.
Score: 0

Accepted Answers:
$\dfrac{\pi\sigma^2 a}{E}$

4) The stress field in terms of Westergaard stress function Z for Mode-I case, with crack centre as origin, is obtained as *1 point*

- $\begin{Bmatrix}\sigma_x\\ \sigma_y\\ \tau_{xy}\end{Bmatrix} = \begin{Bmatrix}ReZ + yImZ\\ ReZ - yImZ\\ -yImZ\end{Bmatrix}$
- $\begin{Bmatrix}\sigma_x\\ \sigma_y\\ \tau_{xy}\end{Bmatrix} = \begin{Bmatrix}ReZ + yImZ'\\ ReZ - yImZ'\\ yImZ'\end{Bmatrix}$
- $\begin{Bmatrix}\sigma_x\\ \sigma_y\\ \tau_{xy}\end{Bmatrix} = \begin{Bmatrix}ReZ' + yImZ\\ ReZ' - yImZ\\ -yImZ\end{Bmatrix}$
- $\begin{Bmatrix}\sigma_x\\ \sigma_y\\ \tau_{xy}\end{Bmatrix} = \begin{Bmatrix}ReZ - yImZ'\\ ReZ + yImZ'\\ -yReZ\end{Bmatrix}$

No, the answer is incorrect.
Score: 0

Accepted Answers:
$\begin{Bmatrix}\sigma_x\\ \sigma_y\\ \tau_{xy}\end{Bmatrix} = \begin{Bmatrix}ReZ - yImZ'\\ ReZ + yImZ'\\ -yReZ\end{Bmatrix}$

5) If $Z(z_0)$ is the Westergaard stress function of the problem, and z_0 is the distance with respect to the crack tip, then Stress intensity factor K is defined by *1 point*

- $K = \lim_{z_0 \to 0} \sqrt{2}Z(z_0)$
- $K = \lim_{z_0 \to 0} \sqrt{z_0}Z(z_0)$
- $K = \lim_{z_0 \to 0} \sqrt{2\pi z_0}$
- $K = \lim_{z_0 \to 0} \sqrt{2\pi z_0}Z(z_0)$

No, the answer is incorrect.
Score: 0

Accepted Answers:
$K = \lim_{z_0 \to 0} \sqrt{2\pi z_0}Z(z_0)$

6) Which feature is captured by the second term modification of the stress field proposed by Irwin? *1 point*

- Variation of τ_{max} along crack axis
- Forward/ Backward tilting of isochromatic fringe from the crack tip
- Variation of τ_{xy} along crack axis
- None of the above

No, the answer is incorrect.
Score: 0

Accepted Answers:
Forward/ Backward tilting of isochromatic fringe from the crack tip

7) The numerical plot of isochromatics by Irwin's two- parameter equation gives fringes that are *1 point*

- symmetric about crack axis
- symmetric about axis perpendicular to crack axis
- symmetric about both of these axes
- asymmetric fringes

No, the answer is incorrect.
Score: 0

Accepted Answers:
symmetric about crack axis

8) Which of the following statements are true regarding modified Westergaard's equation proposed by Tada, Paris and Irwin. *1 point*

A. Cannot be used for long cracks
B. Predicts only a constant fringe order along crack axis
C. Predicts variation of fringe order along crack axis
D. Suitable near the boundaries and stress concentration zones

- A, B, C and D
- A and D
- A and B
- C and D

No, the answer is incorrect.
Score: 0

Accepted Answers:
A and B

9) Generalized Westergaard equation developed by Sanford *1 point*

- Predicts variation of maximum shear stress along the crack axis
- Predicts constant shear stress along crack axis
- Predicts constant shear stress perpendicular to the crack axis
- Predicts variation of in plane shear stress along crack axis

No, the answer is incorrect.
Score: 0

Accepted Answers:
Predicts variation of maximum shear stress along the crack axis

10) Correct relation between energy release rate and SIF is *1 point*

- $G_1 = \frac{K_I^2}{2E}$
- $G_1 = \frac{K_I}{E^2}$
- $G_1 = \frac{K_I^2}{G}$
- $G_1 = \frac{K_I^2}{E}$

No, the answer is incorrect.
Score: 0

Accepted Answers:
$G_1 = \frac{K_I^2}{E}$

11) With crack center as origin, Crack Opening Displacement for Mode I is given by (a is the crack length and x is the crack axis) *1 point*

- $COD = \frac{2\sigma}{E}\sqrt{a^2 - x^2}$
- $COD = \frac{4\sigma}{E}\sqrt{x^2 - a^2}$
- $COD = \frac{4\sigma}{E}\sqrt{a^2 - x^2}$
- $COD = \frac{4\sigma}{G}\sqrt{a^2 - x^2}$

No, the answer is incorrect.
Score: 0

Accepted Answers:
$COD = \frac{4\sigma}{E}\sqrt{a^2 - x^2}$

12) The Airy's stress function for Mode-II crack is $\varphi = -yRe\bar{Z}$ where, $\bar{Z} = \int Zdz$ The Westergaard stress function is $Z = \dfrac{\tau z}{\sqrt{z^2 - a^2}}$. After shifting the origin and making the approximation for near tip field, the displacement field is obtained as . *2 points*

$$\left(\begin{aligned} k &= \tfrac{3-v}{1+v}\ for\ plane\ stress \\ &= 3 - 4v\ for\ plane\ strain \end{aligned} \right)$$

- $u_x = \sqrt{\frac{r}{2\pi}}\sin\frac{\theta}{2}\left[k + 1 + 2\cos^2\frac{\theta}{2}\right]$
  $u_y = \sqrt{\frac{r}{2\pi}}\cos\frac{\theta}{2}\left[1 - k + 2\sin^2\frac{\theta}{2}\right]$
- $u_x = \frac{K_{II}}{2G}\sqrt{\frac{r}{2\pi}}\sin\frac{\theta}{2}[k + 1]$
  $u_y = \frac{K_{II}}{2G}\sqrt{\frac{r}{2\pi}}\cos\frac{\theta}{2}[1 - k]$
- $u_x = \frac{K_{II}}{2G}\sqrt{\frac{r}{2\pi}}\left[2\cos^2\frac{\theta}{2}\right]$
  $u_y = \frac{K_{II}}{2G}\sqrt{\frac{r}{2\pi}}\left[2\sin^2\frac{\theta}{2}\right]$
- $u_x = \frac{K_{II}}{2G}\sqrt{\frac{r}{2\pi}}\sin\frac{\theta}{2}\left[k + 1 + 2\cos^2\frac{\theta}{2}\right]$
  $u_y = \frac{K_{II}}{2G}\sqrt{\frac{r}{2\pi}}\cos\frac{\theta}{2}\left[1 - k + 2\sin^2\frac{\theta}{2}\right]$

No, the answer is incorrect.
Score: 0

Accepted Answers:
$u_x = \frac{K_{II}}{2G}\sqrt{\frac{r}{2\pi}}\sin\frac{\theta}{2}\left[k + 1 + 2\cos^2\frac{\theta}{2}\right]$
$u_y = \frac{K_{II}}{2G}\sqrt{\frac{r}{2\pi}}\cos\frac{\theta}{2}\left[1 - k + 2\sin^2\frac{\theta}{2}\right]$

The Airy's stress function for Mode-I crack problem is $\phi = Re\bar{\bar{Z}} + yIm\bar{Z}$ where, $\bar{Z} = \int Zdz$; $\bar{\bar{Z}} = \int \bar{Z}dz$. For this problem, choose the correct option.

Based on the above data answer the following questions 13-15 :

13) After shifting the origin to the crack tip and making the approximation for near tip field, Z is given as *1 point*

- $Z = \dfrac{\sigma a}{\sqrt{2az_0}}$
- $Z = \dfrac{\sigma a}{\sqrt{2z_0}}$
- $Z = \dfrac{\sigma}{\sqrt{2az_0}}$
- $Z = \dfrac{\sigma a}{\sqrt{az_0}}$

No, the answer is incorrect.
Score: 0

Accepted Answers:
$Z = \dfrac{\sigma a}{\sqrt{2az_0}}$

14) The stress field is given as *2 points*

- $\begin{Bmatrix}\sigma_x\\ \sigma_y\\ \tau_{xy}\end{Bmatrix} = \frac{K_I}{\sqrt{2\pi r}}\cos\frac{\theta}{2}\begin{Bmatrix}\sin\frac{\theta}{2}\sin\frac{3\theta}{2}\\ \sin\frac{\theta}{2}\sin\frac{3\theta}{2}\\ \sin\frac{\theta}{2}\sin\frac{3\theta}{2}\end{Bmatrix}$
- $\begin{Bmatrix}\sigma_x\\ \sigma_y\\ \tau_{xy}\end{Bmatrix} = \frac{K_I}{\sqrt{2\pi r}}\cos\frac{\theta}{2}\begin{Bmatrix}1 - \sin\frac{\theta}{2}\\ 1 + \sin\frac{\theta}{2}\\ \sin\frac{\theta}{2}\end{Bmatrix}$
- $\begin{Bmatrix}\sigma_x\\ \sigma_y\\ \tau_{xy}\end{Bmatrix} = \frac{K_I}{\sqrt{2\pi r}}\cos\frac{\theta}{2}\begin{Bmatrix}1 - \sin\frac{\theta}{2}\sin\frac{3\theta}{2}\\ 1 + \sin\frac{\theta}{2}\sin\frac{3\theta}{2}\\ \sin\frac{\theta}{2}\sin\frac{3\theta}{2}\end{Bmatrix}$
- $\begin{Bmatrix}\sigma_x\\ \sigma_y\\ \tau_{xy}\end{Bmatrix} = \cos\frac{\theta}{2}\begin{Bmatrix}1 - \sin\frac{\theta}{2}\sin\frac{3\theta}{2}\\ 1 + \sin\frac{\theta}{2}\sin\frac{3\theta}{2}\\ \sin\frac{\theta}{2}\sin\frac{3\theta}{2}\end{Bmatrix}$

No, the answer is incorrect.
Score: 0

Accepted Answers:
$\begin{Bmatrix}\sigma_x\\ \sigma_y\\ \tau_{xy}\end{Bmatrix} = \frac{K_I}{\sqrt{2\pi r}}\cos\frac{\theta}{2}\begin{Bmatrix}1 - \sin\frac{\theta}{2}\sin\frac{3\theta}{2}\\ 1 + \sin\frac{\theta}{2}\sin\frac{3\theta}{2}\\ \sin\frac{\theta}{2}\sin\frac{3\theta}{2}\end{Bmatrix}$

15) The displacement field for plane stress Mode-I condition is given by *2 points*

- $u = \frac{1}{2G}\left[\frac{1-v}{1+v}Re\bar{Z}_1 - yIm\bar{Z}_1\right]; v = \frac{1}{2G}\left[\frac{2}{1+v}RImZ_1 - yRe\bar{Z}_1\right]$
- $u = \frac{1}{2G}\left[\frac{1-v}{1+v}Re\bar{Z}_1 - yImZ_1\right]; v = \frac{1}{2G}\left[\frac{2}{1+v}Im\bar{Z}_1 - yReZ_1\right]$
- $u = \frac{1}{2G}\left[\frac{1-v}{1+v}Im\bar{Z}_1 - yReZ_1\right]; v = \frac{1}{2G}\left[\frac{2}{1+v}Im\bar{Z}_1 - yReZ_1\right]$
- $u = \frac{1}{G}\left[\frac{1-v}{1+v}Re\bar{Z}_1 - yImZ_1\right]; v = \frac{1}{G}\left[\frac{2}{1+v}Im\bar{Z}_1 - yReZ_1\right]$

No, the answer is incorrect.
Score: 0

Accepted Answers:
$u = \frac{1}{2G}\left[\frac{1-v}{1+v}Re\bar{Z}_1 - yImZ_1\right]; v = \frac{1}{2G}\left[\frac{2}{1+v}Im\bar{Z}_1 - yReZ_1\right]$