Assignment B

Due at 11:59 PM on Sunday, 2011-03-20.

1. The function $f(n)$ is defined recursively as follows:
   
   $f(n) = \begin{cases} 
   1 & \text{if } n = 1 \\
   f(n-1) + f(n-2) & \text{if } n > 1 
   \end{cases}$

   a. Calculate $f(5)$.
   b. Show that $f(n)$ is the $n$-th Fibonacci number.

2. Let $a$ and $b$ be real numbers such that $a + b = 3$ and $ab = 2$. Compute
   $a^2 + b^2$.

3. If $f(x) = x^3 - 3x^2 + a$ and $f(2) = 0$, find the value of $a$.

4. In a class of 100 students, 60 students take math, 50 students take science, and 40 students take both math and science. How many students take neither math nor science?

5. If $A$ and $B$ are $2 \times 2$ matrices and $A = \begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix}$, $B = \begin{pmatrix} 5 & 6 \\ 7 & 8 \end{pmatrix}$, find $AB$.

6. A tree has 100 branches. If each branch has 5 leaves, how many leaves are there in total?

7. Let $f(x) = \frac{1}{x}$. Compute $\int_{1}^{2} f(x) dx$.

8. A company has two types of products: product A and product B. The company sells 100 units of product A and 200 units of product B. If the profit margin for product A is 10% and for product B is 20%, calculate the total profit.

9. Consider the sequence defined by $a_1 = 2$, $a_2 = 3$, and $a_{n+2} = a_{n+1} + a_n$ for $n \geq 1$. Find the 10th term of the sequence.

10. Solve the equation $2x + 3 = 7$ for $x$.

11. Simplify the expression $\frac{3x^2 - 9x}{x^2 - 9}$.

12. A rectangle has a length of 10 units and a width of 5 units. Find the area and the perimeter of the rectangle.

13. If $\sin(\theta) = \frac{1}{2}$, find the value of $\cos(\theta)$.

14. If $f(x) = \sin(x)$ and $g(x) = \cos(x)$, find the derivative of $f(x) + g(x)$.

15. The probability of rain on any given day is 0.3. What is the probability that it does not rain on 3 consecutive days?

16. The area of a circle is given by $A = \pi r^2$. If the radius of a circle is 4 units, what is the area of the circle?

17. The volume of a sphere is given by $V = \frac{4}{3} \pi r^3$. If the radius of a sphere is 3 units, what is the volume of the sphere?