Assignment 5

Due on 08/06/21 at 11:59 PM

1. A two-dimensional matrix $A$ with entries $a_{ij}$ is defined by $a_{ij} = i + j$ for $i, j = 1, 2, 3$. Determine the determinant of $A$.

2. A three-dimensional matrix $B$ with entries $b_{ijk} = i + j + k$ for $i, j, k = 1, 2, 3$ is defined. Find the trace of $B$.

3. A linear transformation $T: \mathbb{R}^2 \to \mathbb{R}^2$ is defined by $T(x, y) = (2x - y, x + 2y)$. Find the matrix representation of $T$ with respect to the standard basis.

4. Consider a three-dimensional linear transformation $S: \mathbb{R}^3 \to \mathbb{R}^3$ with matrix representation $S = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 3 \end{pmatrix}$. Find the image of the vector $v = (1, 2, 3)$ under $S$.

5. A linear transformation $R: \mathbb{R}^2 \to \mathbb{R}^2$ is defined by $R(x, y) = (x - y, x + y)$. Find the matrix representation of $R$ with respect to the standard basis.

6. A matrix $M$ is defined by $M = \begin{pmatrix} 2 & 1 \\ 1 & 3 \end{pmatrix}$. Find the eigenvalues and eigenvectors of $M$. How do they relate to the matrix $M$?

7. A linear transformation $T: \mathbb{R}^3 \to \mathbb{R}^3$ is defined by $T(x, y, z) = (2x - y + z, x + 2y - z, 3x + y - z)$. Find the matrix representation of $T$ with respect to the standard basis.

8. A linear transformation $S: \mathbb{R}^2 \to \mathbb{R}^2$ is defined by $S(x, y) = (x + y, x - y)$. Find the matrix representation of $S$ with respect to the basis $\{(1, 0), (0, 1)\}$.

9. A linear transformation $R: \mathbb{R}^3 \to \mathbb{R}^3$ is defined by $R(x, y, z) = (x - y + z, x + y - z, x + y + z)$. Find the matrix representation of $R$ with respect to the standard basis.

10. A linear transformation $T: \mathbb{R}^2 \to \mathbb{R}^2$ is defined by $T(x, y) = (2x, y)$. Find the matrix representation of $T$ with respect to the basis $\{(1, 0), (0, 1)\}$.