Assignment 3

3.1 For the taut string carrying a point mass $m$ as shown in figure, use the Ritz method to discretize the system assuming the transverse displacement field variable $w(x,t) = a(t)x(l-x)$. Find the fundamental natural frequency of the discrete system if $m/\rho Al \to 0$.

![Figure 1: Problem 1](image)

(a) $\omega_1 = 3.1623 \sqrt{\frac{T}{\rho Al^2}}$

(b) $\omega_1 = 3.1416 \sqrt{\frac{T}{\rho Al^2}}$

(c) $\omega_1 = 2.3094 \sqrt{\frac{T}{\rho Al^2}}$

(d) $\omega_1 = 2.9153 \sqrt{\frac{T}{\rho Al^2}}$

3.2 The vibrations of a fixed-fixed taut string of length $l$, mass density $\rho$, cross-sectional area $A$, under tension $T$ is described by a deflection field variable $w(x,t)$. Assuming $w(x,t) = a_1(t)x(l-x) + a_2(t)x^2(l-x)$, discretize the system and obtain the first two fundamental natural frequencies of the discretized system.

(a) $\omega_1 = 3.1416 \sqrt{\frac{T}{\rho Al^2}}$ and $\omega_2 = 6.2832 \sqrt{\frac{T}{\rho Al^2}}$

(b) $\omega_1 = 3.25 \sqrt{\frac{T}{\rho Al^2}}$ and $\omega_2 = 6.5 \sqrt{\frac{T}{\rho Al^2}}$

(c) $\omega_1 = 3.1924 \sqrt{\frac{T}{\rho Al^2}}$ and $\omega_2 = 6.3548 \sqrt{\frac{T}{\rho Al^2}}$

(d) $\omega_1 = 3.1623 \sqrt{\frac{T}{\rho Al^2}}$ and $\omega_2 = 6.4807 \sqrt{\frac{T}{\rho Al^2}}$

3.3 A homogeneous tapered bar of circular cross-section is shown in figure. Assume the admissible function for the field variable (longitudinal displacement of the bar) of the form $H_k(x) = (x/l)^k$, where $k$ is an integer. Using Rayleigh’s quotient, determine the value of $k$ that yields the lowest value of the fundamental natural frequency of the system.

(a) $k = 0.5$

(b) $k = 1.0431$
3.4 Use Galerkin’s method to discretize the equation of motion of a hanging string taking the comparison functions as $P_i(x) = x^i$, $i = 1, 2$. Determine the two fundamental eigen frequencies for the discretized system.

(a) $\omega_1 = 1.1542\sqrt{\frac{g}{l}}$ and $\omega_2 = 3.1154\sqrt{\frac{g}{l}}$

(b) $\omega_1 = 1.2025\sqrt{\frac{g}{l}}$ and $\omega_2 = 3.0365\sqrt{\frac{g}{l}}$

(c) $\omega_1 = 1.4642\sqrt{\frac{g}{l}}$ and $\omega_2 = 3.8413\sqrt{\frac{g}{l}}$

(d) $\omega_1 = 1.5708\sqrt{\frac{g}{l}}$ and $\omega_2 = 3.1416\sqrt{\frac{g}{l}}$

(c) $k = 0.3355$

(d) $k = 1.25$