Unit 12 - Week 9: Two dimensional Scalar and Vector field problems

Course outline

- How does an NPTEL online course work?
- MATLAB
- Week 0: Prerequisite
- Week 1: Variational Calculus and Minimization Problem
- Week 2: One dimensional Finite Element Analysis
- Week 3: Structural Elements in One Dimensional FEM
- Week 4: Structural Elements in One Dimensional FEM
- Week 5: Structural Elements in One Dimensional FEM, and Generalized One Dimensional Finite Element Code in Computer Programming
- Week 6: Brief Background of Tensor Calculus
- Week 7: Two dimensional Scalar field problems
- Week 8: Two dimensional Scalar field problems
- Week 9: Two dimensional Scalar and Vector field problems
  - ○ Lec 26: Demonstration of the MATLAB code
  - ● Lec 27: Elasticity problems in two dimension and obtaining the weak form
  - ● Lec 28: Deriving element stiffness matrix and element force vector, numerical example
  - ○ Quiz : Assignment 9
  - ○ Feedback Form
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- Week 10: Two dimensional Vector field and Eigen value problems
- Week 11:Eigen value problems and Transient problem in 1D & 2D Scalar Valued Problems
- Week 12: FEM formulation for 3D Elastic problem and challenges
- Live session: Dr. Atanu Banerjee, Date : 16/12/2020 Time : 3:15:00 PM

Assignment 9

The due date for submitting this assignment has passed.
As per our records you have not submitted this assignment.

Due on 2020-11-18, 23:59 IST.

Q1. Form the connectivity matrix for the meshed polygonal domain (1-2-3-4-5-6) shown in Fig.1. Denote the $(i,j)^{th}$ entry of the element-stiffness matrix of element (m) (element number in **blue**) as $K_{ij}^{(m)}$; and i^{th} entry of the element force vector of element (m) as $F_i^{(m)}$. The global node numbers are shown in **black**, whereas local node numbers are illustrated in **red**. Use only the numbering scheme shown in Fig.1 and answer the following questions 1-8.

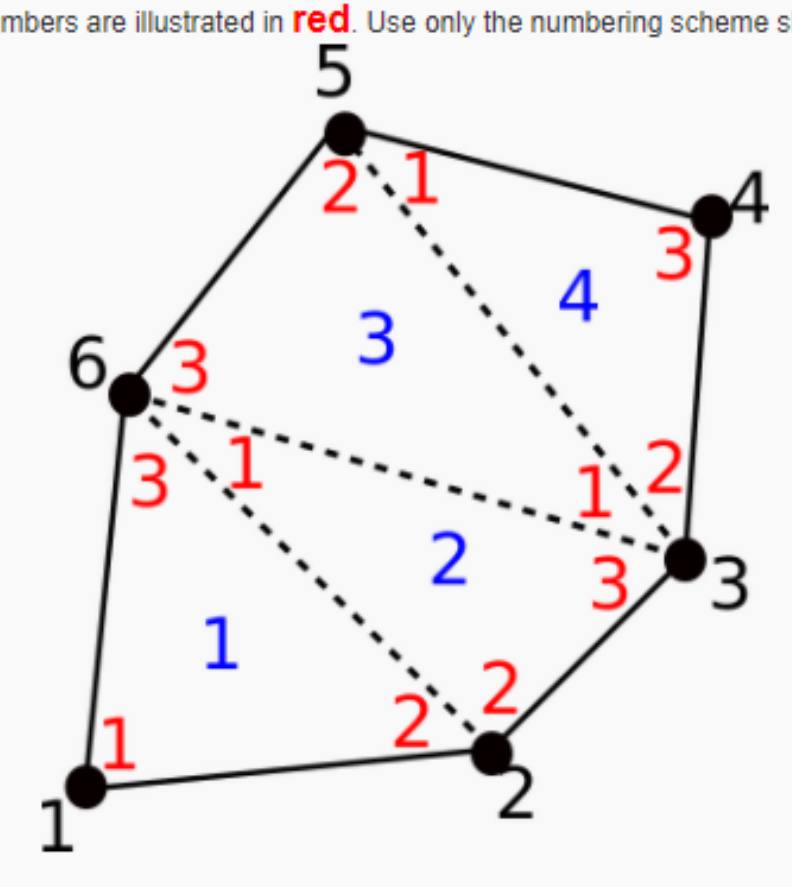


Figure 1: Discretized domain

1) The global stiffness entry K_{11} , can be written in terms of the elements' contribution as, *2 points*

○ (a) $K_{11}^{(1)} + K_{22}^{(2)}$
○ (b) $K_{11}^{(1)}$
○ (c) $K_{31}^{(1)} + K_{22}^{(2)}$
○ (d) $K_{22}^{(3)}$

No, the answer is incorrect.
Score: 0
Accepted Answers:
(b) $K_{11}^{(1)}$

2) The global stiffness entry K_{22} , can be written in terms of the elements' contribution as, *2 points*

○ (a) $K_{22}^{(3)}$
○ (b) $K_{12}^{(1)} + K_{22}^{(2)}$
○ (c) $K_{31}^{(1)}$
○ (d) $K_{22}^{(1)} + K_{22}^{(2)}$

No, the answer is incorrect.
Score: 0
Accepted Answers:
(d) $K_{22}^{(1)} + K_{22}^{(2)}$

3) The global stiffness entry K_{33} , can be written in terms of the elements' contribution as, *2 points*

○ (a) $K_{22}^{(3)}$
○ (b) $K_{33}^{(2)} + K_{11}^{(5)} + K_{22}^{(1)}$
○ (c) $K_{13}^{(2)} + K_{11}^{(3)} + K_{22}^{(4)}$
○ (d) $K_{33}^{(2)} + K_{11}^{(3)} + K_{22}^{(4)}$

No, the answer is incorrect.
Score: 0
Accepted Answers:
(d) $K_{33}^{(2)} + K_{11}^{(3)} + K_{22}^{(4)}$

4) The global stiffness entry K_{35} , can be written in terms of the elements' contribution as, *2 points*

○ (a) $K_{12}^{(3)} + K_{21}^{(4)}$
○ (b) $K_{33}^{(2)} + K_{11}^{(5)} + K_{22}^{(1)}$
○ (c) $K_{13}^{(2)} + K_{11}^{(3)} + K_{22}^{(4)}$
○ (d) $K_{22}^{(3)}$

No, the answer is incorrect.
Score: 0
Accepted Answers:
(a) $K_{12}^{(3)} + K_{21}^{(4)}$

5) The global stiffness entry K_{56}, can be written in terms of the elements' contribution as, *2 points*

○ (a) $K_{23}^{(3)}$
○ (b) $K_{33}^{(3)} + K_{22}^{(3)}$
○ (c) $K_{33}^{(3)}$
○ (d) $K_{22}^{(3)}$

No, the answer is incorrect.
Score: 0
Accepted Answers:
(a) $K_{23}^{(3)}$

6) The global stiffness entry K_{13} , can be written in terms of the elements' contribution as, *2 points*

○ (a) $K_{22}^{(3)}$
○ (b) $K_{33}^{(2)} + K_{11}^{(5)} + K_{22}^{(1)}$
○ (c) $K_{13}^{(2)} + K_{11}^{(3)} + K_{22}^{(4)}$
○ (d) 0

No, the answer is incorrect.
Score: 0
Accepted Answers:

(d) 0

7) The global stiffness entry K_{55}, can be written in terms of the elements' contribution as, *2 points*

○ (a) $K_{13}^{(2)} + K_{11}^{(3)} + K_{22}^{(4)}$
○ (b) $K_{33}^{(2)} + K_{11}^{(5)} + K_{22}^{(1)}$
○ (c) $K_{22}^{(3)} + K_{11}^{(4)}$
○ (d) $K_{22}^{(3)}$

No, the answer is incorrect.
Score: 0
Accepted Answers:
(c) $K_{22}^{(3)} + K_{11}^{(4)}$

8) The global force entry F_6 can be written in terms of the elements' contribution as *2 points*

○ (a) $F_3^{(2)} + F_2^{(1)} + F_3^{(3)}$
○ (b) $F_3^{(1)} + F_1^{(2)} + F_3^{(3)}$
○ (c) $F_3^{(1)} + F_2^{(2)} + F_3^{(3)}$
○ (d) $F_2^{(3)}$

No, the answer is incorrect.
Score: 0
Accepted Answers:
(b) $F_3^{(1)} + F_1^{(2)} + F_3^{(3)}$

Q2. Find out the exact expression of the following shape functions. Answer question 9-16.

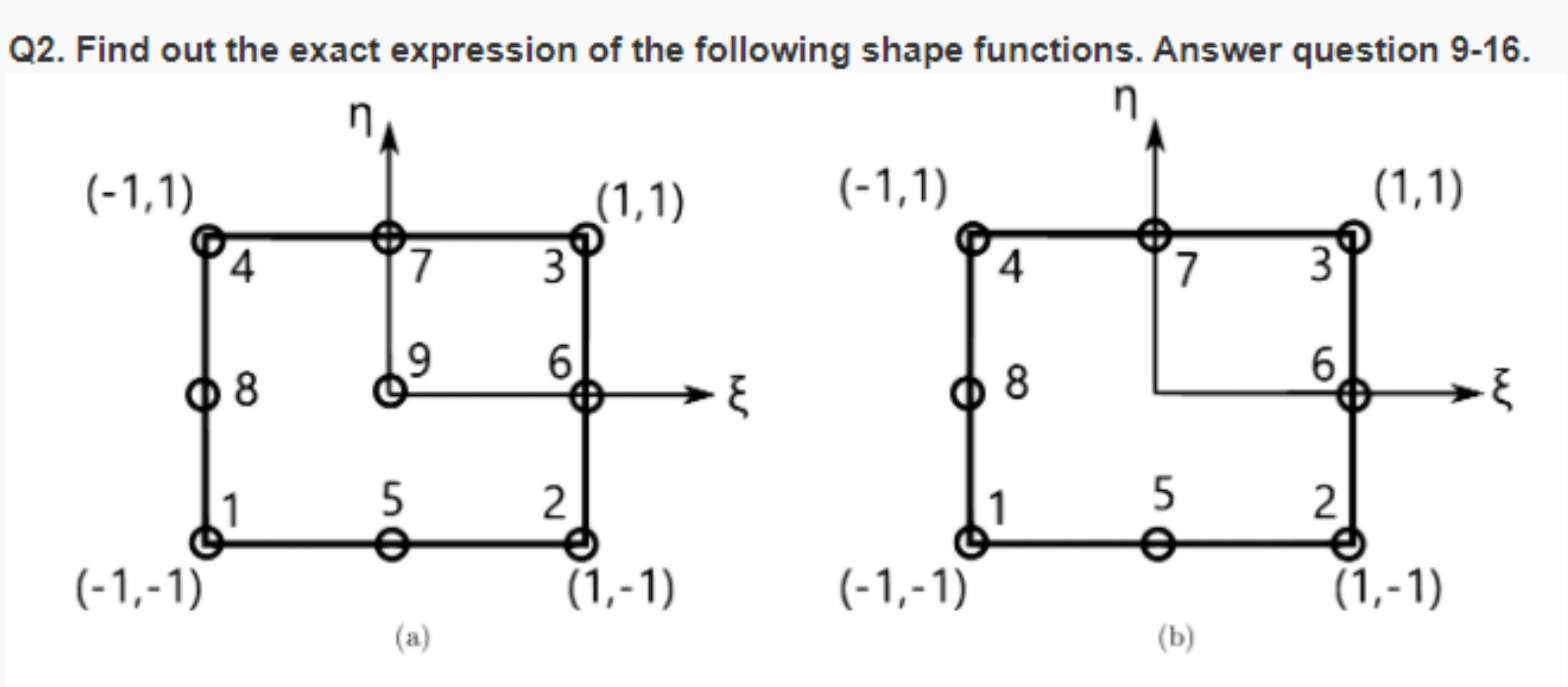


Figure 2

9) The expression of the shape function $\hat{N}_2$ for the quadratic Lagrange element, shown in Fig. 2a is, *2 points*

○ (a) $\frac{1}{4}\xi\eta(\xi+1)(\eta-1)$
○ (b) $\frac{1}{2}\xi\eta(\xi+1)(\eta-1)$
○ (c) $\frac{1}{4}\xi\eta(\xi-1)(\eta-1)$
○ (d) $\frac{1}{4}\xi\eta(\xi+1)(\eta+1)$

No, the answer is incorrect.
Score: 0
Accepted Answers:
(a) $\frac{1}{4}\xi\eta(\xi+1)(\eta-1)$

10) The expression of the shape function $\hat{N}_5$ for the quadratic Lagrange element, shown in Fig. 2a is *2 points*

○ (a) $\frac{1}{2}(\xi-1)(\xi+1)\eta(\eta-1)$
○ (b) $\frac{1}{2}(\xi+1)(\xi+1)\eta(\eta-1)$
○ (c) $\frac{1}{2}(\xi-1)(\xi+1)\eta(\eta+1)$
○ (d) $-\frac{1}{2}(\xi-1)(\xi+1)\eta(\eta-1)$

No, the answer is incorrect.
Score: 0
Accepted Answers:
(d) $-\frac{1}{2}(\xi-1)(\xi+1)\eta(\eta-1)$

11) The expression of the shape function $\hat{N}_7$ for the quadratic Lagrange element, shown in Fig. 2a is *2 points*

○ (a) $-\frac{1}{2}(\xi-1)(\xi+1)\eta(\eta+1)$
○ (b) $\frac{1}{2}(\xi-1)(\xi+1)\eta(\eta+1)$
○ (c) $-\frac{1}{2}(\xi+1)(\xi+1)\eta(\eta+1)$
○ (d) $-\frac{1}{2}(\xi-1)(\xi+1)\eta(\eta-1)$

No, the answer is incorrect.
Score: 0
Accepted Answers:
(a) $-\frac{1}{2}(\xi-1)(\xi+1)\eta(\eta+1)$

12) The expression of the shape function $\hat{N}_9$ for the quadratic Lagrange element, shown in Fig. 2a is *2 points*

○ (a) $\xi(\xi-1)(\eta-1)(\eta+1)$
○ (b) $(\xi+1)(\xi-1)(\eta-1)(\eta+1)$
○ (c) $\xi(\xi+1)(\eta-1)(\eta+1)$
○ (d) $(\xi+1)(\xi-1)\eta(\eta+1)$

No, the answer is incorrect.
Score: 0
Accepted Answers:
(b) $(\xi+1)(\xi-1)(\eta-1)(\eta+1)$

13) The expression of the shape function $\hat{N}_1$ for the quadratic Serendipity element, shown in Fig. 2b is *2 points*

○ (a) $-\frac{1}{4}(\xi-1)(\eta-1)(\xi+\eta+1)$
○ (b) $-\frac{1}{4}(\xi-1)(\eta-1)(\xi+\eta-1)$
○ (c) $-\frac{1}{2}(\xi-1)(\eta-1)(\xi+\eta+1)$
○ (d) $-\frac{1}{2}(\xi-1)(\eta-1)(\xi+\eta-1)$

No, the answer is incorrect.
Score: 0
Accepted Answers:
(a) $-\frac{1}{4}(\xi-1)(\eta-1)(\xi+\eta+1)$

14) The expression of the shape function $\hat{N}_5$ for the quadratic Serendipity element, shown in Fig. 2b is *2 points*

○ (a) $\frac{1}{4}(\xi+1)(\xi-1)(\eta-1)$
○ (b) $\frac{1}{2}(\xi+1)(\xi-1)(\eta+1)$
○ (c) $\frac{1}{2}(\xi+1)(\xi-1)(\eta-1)$
○ (d) $(\xi+1)(\xi-1)(\eta-1)$

No, the answer is incorrect.
Score: 0
Accepted Answers:
(c) $\frac{1}{2}(\xi+1)(\xi-1)(\eta-1)$

15) The expression of the shape function $\hat{N}_6$ for the quadratic Serendipity element, shown in Fig. 2b is *2 points*

○ (a) $\frac{1}{4}(\xi+1)(\eta+1)(\eta-1)$
○ (b) $-\frac{1}{2}(\eta-1)(\eta+1)(\xi+1)$
○ (c) $\frac{1}{2}(\xi+1)(\xi-1)(\eta-1)$
○ (d) $(\xi+1)(\xi-1)(\eta-1)$

No, the answer is incorrect.
Score: 0
Accepted Answers:
(b) $-\frac{1}{2}(\eta-1)(\eta+1)(\xi+1)$

16) The expression of the shape function $\hat{N}_8$ for the quadratic Serendipity element, shown in Fig. 2b is *2 points*

○ (a) $\frac{1}{4}(\xi+1)(\xi-1)(\eta-1)$
○ (b) $\frac{1}{2}(\xi-1)(\eta+1)(\eta-1)$
○ (c) $\frac{1}{2}(\xi+1)(\xi-1)(\eta-1)$
○ (d) $(\xi+1)(\xi-1)(\eta-1)$

No, the answer is incorrect.
Score: 0
Accepted Answers:
(b) $\frac{1}{2}(\xi-1)(\eta+1)(\eta-1)$

Q3. As shown in Fig. 3, a physical element (τ) has been linearly mapped to the master element ($\hat{\tau}$). The coordinates of the vertices are shown in the figure itself. Answer question 17.

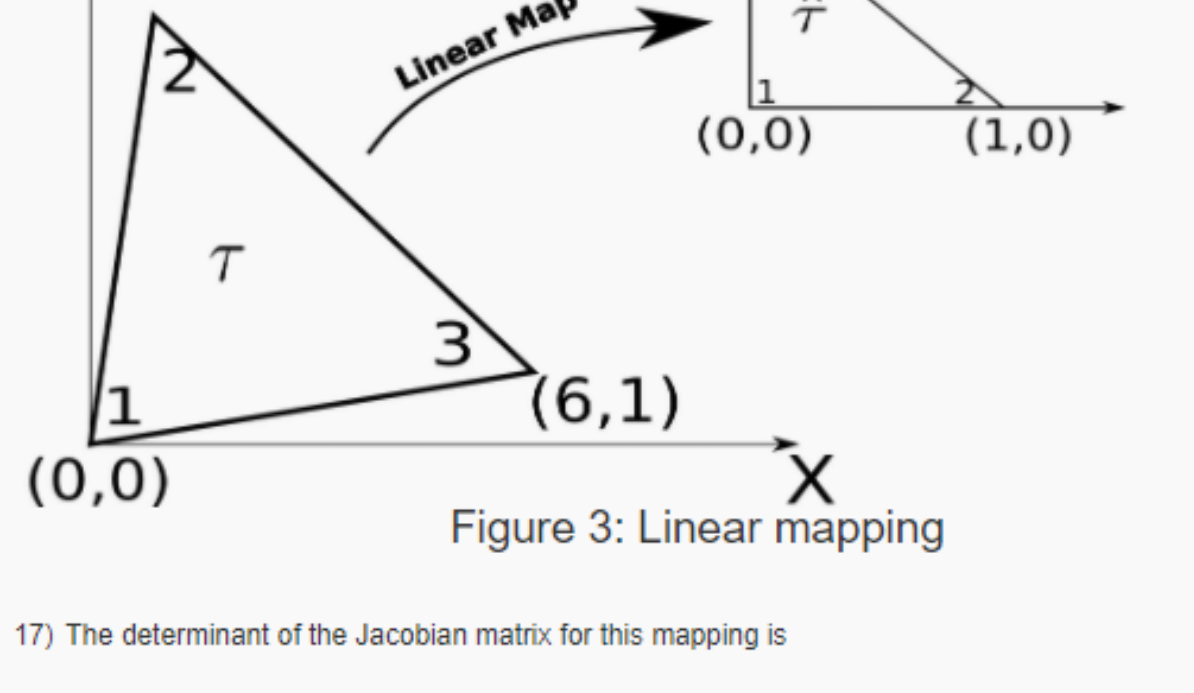


Figure 3: Linear mapping

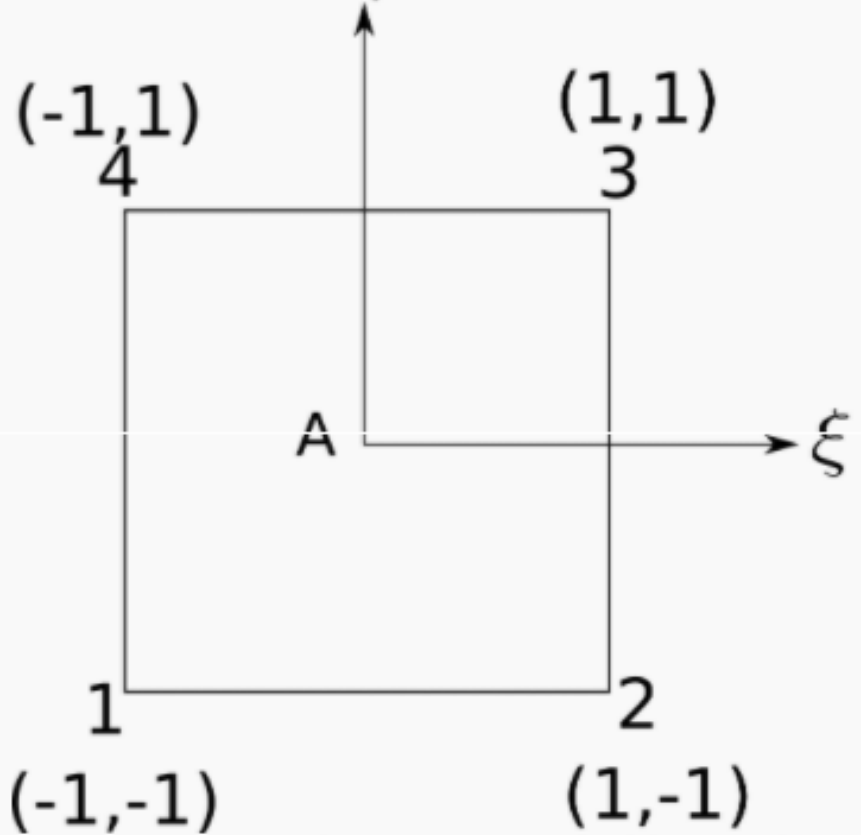
17) The determinant of the Jacobian matrix for this mapping is

Hint

No, the answer is incorrect.
Score: 0
Accepted Answers:
(Type: Range) -40.1,-39.9

4 points

Q4. Write a small code for numerical integration following Gauss Quadrature rule, and answer the questions 18-20 below referring Fig.4.



$$N_1 = \frac{1}{4}(1-\xi)(1-\eta)$$

$$N_2 = \frac{1}{4}(1+\xi)(1-\eta)$$

Figure 4: Numerical integration

18) Using 1 point Gauss Quadrature rule, in each direction, calculate the value of $\int_A N_1 N_2 d\xi d\eta$= *2 points*

○ (a) 1/4
○ (b) 1/2
○ (c) 1/5
○ (d) 1/6

No, the answer is incorrect.
Score: 0
Accepted Answers:
(a) 1/4

19) Using 2 point Gauss Quadrature rule, in each direction, calculate the value of $\int_A N_1 N_2 d\xi d\eta$= *2 points*

○ (a) 2/9
○ (b) 1/9
○ (c) 3/9
○ (d) 4/9

No, the answer is incorrect.
Score: 0
Accepted Answers:
(a) 2/9

20) Using 3 point Gauss Quadrature rule, in each direction, calculate the value of $\int_A N_1 N_2 d\xi d\eta$ = *2 points*

○ (a) 1
○ (b) 2
○ (c) 1/9
○ (d) 2/9

No, the answer is incorrect.
Score: 0
Accepted Answers:
(d) 2/9