

Unit 15 - Week 12: FEM formulation for 3D Elastic problem and challenges

Course outline
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MATLAB
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Week 1: Variational Calculus and Minimization Problem
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Week 3: Structural Elements in One Dimensional FEM
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Week 12: FEM formulation for 3D Elastic problem and challenges
<ul style="list-style-type: none"> Lec 35 Solving elasticity problems in 3D using FEM, Solvers Quiz : Assignment 12 Feedback form Assignment 12 solution
Live session: Dr. Atanu Banerjee, Date : 16/12/2020 Time : 3:15:00 PM

Assignment 12

The due date for submitting this assignment has passed. **Due on 2020-12-09, 23:59 IST.**
As per our records you have not submitted this assignment.

Consider an unsteady heat conduction problem in an insulated bar of length $l = 1$ as defined below. Assume one quadratic bar element for spatial discretization, and Crank-Nicolson scheme for time discretization to calculate the temperature at the mid point of the domain as time varies. Assume the heat generation term as $g = 1$. You may take help of MatLab for obtaining the solution.

$$\frac{\partial^2 u}{\partial x^2} + g = \frac{\partial u}{\partial t} \quad \text{for } 0 < x < 1$$

$$u(0, t) = u(1, t) = 0, \quad (\text{Boundary condition})$$

$$u(x, 0) = 0, \quad (\text{Initial condition})$$

1) Choose the correct weak form. 2 points

- (a) $\int_0^1 \frac{\partial u}{\partial x} \frac{\partial w}{\partial x} dx = \int_0^1 g w dx + w \frac{\partial u}{\partial x} \Big|_0^1$
- (b) $\int_0^1 \frac{\partial u}{\partial x} w dx + \int_0^1 \frac{\partial u}{\partial x} \frac{\partial w}{\partial x} dx = \int_0^1 g w dx + w \frac{\partial u}{\partial x} \Big|_0^1$
- (c) $\int_0^1 \frac{\partial u}{\partial x} u dx + \int_0^1 \frac{\partial u}{\partial x} \frac{\partial w}{\partial x} dx = \int_0^1 g w dx + w \frac{\partial u}{\partial x} \Big|_0^1$
- (d) $\int_0^1 \frac{\partial u}{\partial x} w dx + \int_0^1 \frac{\partial u}{\partial x} \frac{\partial w}{\partial x} dx = \int_0^1 g w dx - w \frac{\partial u}{\partial x} \Big|_0^1$

No, the answer is incorrect. Score: 0
Accepted Answers: (b) $\int_0^1 \frac{\partial u}{\partial x} w dx + \int_0^1 \frac{\partial u}{\partial x} \frac{\partial w}{\partial x} dx = \int_0^1 g w dx + w \frac{\partial u}{\partial x} \Big|_0^1$ 2 points

2) Substituting $u = \sum \alpha_j \phi_j$ and $w = \phi_j$ in the weak form one obtains, $M_{ij} \alpha_j + K_{ij} \alpha_j = F_i$, where $M_{ij} = \dots$

- (a) $M_{ij} = \int_0^1 \phi_i \phi_j dx$
- (b) $M_{ij} = \int_0^1 \phi_i \phi_{j,x} dx$
- (c) $M_{ij} = \int_0^1 \phi_i \phi_j dx$
- (d) $M_{ij} = \int_0^1 \phi_{i,x} \phi_j dx$

No, the answer is incorrect. Score: 0
Accepted Answers: (c) $M_{ij} = \int_0^1 \phi_i \phi_j dx$ 2 points

3) Similarly, K_{ij} represents,

- (a) $K_{ij} = \int_0^1 \phi_{i,x} \phi_{j,x} dx$
- (b) $K_{ij} = \int_0^1 \phi_i \phi_{j,x} dx$
- (c) $K_{ij} = \int_0^1 \phi_{i,x} \phi_j dx$
- (d) $K_{ij} = \int_0^1 \phi_i \phi_j dx$

No, the answer is incorrect. Score: 0
Accepted Answers: (a) $K_{ij} = \int_0^1 \phi_{i,x} \phi_{j,x} dx$ 2 points

4) Similarly, F_i represents,

- (a) $F_i = \int_0^1 g \phi_i dx - \frac{\partial u}{\partial x} \phi_i \Big|_0^1$
- (b) $F_i = \int_0^1 g \phi_i dx + \frac{\partial u}{\partial x} \phi_i \Big|_0^1$
- (c) $F_i = \int_0^1 g \phi_i dx - \frac{\partial u}{\partial x} \phi_{i,x} \Big|_0^1$
- (d) $F_i = \int_0^1 g \phi_i dx + \frac{\partial u}{\partial x} \phi_{i,x} \Big|_0^1$

No, the answer is incorrect. Score: 0
Accepted Answers: (b) $F_i = \int_0^1 g \phi_i dx + \frac{\partial u}{\partial x} \phi_i \Big|_0^1$ 2 points

5) The definition of the shape function N_1 (Lagrange shape function for degree $p=2$) in the physical element is,

- (a) $N_1 = \frac{2}{\rho} (x - 1/2)(x - 1)$
- (b) $N_1 = \frac{x^2}{\rho} (x - 1)$
- (c) $N_1 = \frac{x^2}{\rho} (x + 1)$
- (d) $N_1 = \frac{2}{\rho} (x)(x - 1/2)$

No, the answer is incorrect. Score: 0
Accepted Answers: (a) $N_1 = \frac{2}{\rho} (x - 1/2)(x - 1)$ 2 points

6) The definition of the shape function N_2 (Lagrange shape function for degree $p=2$) in the physical element is,

- (a) $N_2 = \frac{2}{\rho} (x - 1/2)(x - 1)$
- (b) $N_2 = \frac{x^2}{\rho} (x - 1)$
- (c) $N_2 = \frac{x^2}{\rho} (x + 1)$
- (d) $N_2 = \frac{2}{\rho} (x)(x - 1/2)$

No, the answer is incorrect. Score: 0
Accepted Answers: (b) $N_2 = \frac{x^2}{\rho} (x - 1)$ 2 points

7) The definition of the shape function N_3 (Lagrange shape function for degree $p=2$) in the physical element is,

- (a) $N_3 = \frac{2}{\rho} (x - 1/2)(x - 1)$
- (b) $N_3 = \frac{x^2}{\rho} (x - 1)$
- (c) $N_3 = \frac{x^2}{\rho} (x + 1)$
- (d) $N_3 = \frac{2}{\rho} (x)(x - 1/2)$

No, the answer is incorrect. Score: 0
Accepted Answers: (b) $N_3 = \frac{x^2}{\rho} (x - 1/2)$ 2 points

8) The size of the so called stiffness matrix, [K], for this problem is,

- (a) 4x4
- (b) 6x6
- (c) 2x2
- (d) 3 x 3

No, the answer is incorrect. Score: 0
Accepted Answers: (d) 3 x 3 2 points

9) During time discretization we use, $(1 - \mu)\alpha^{(n)} + \mu\alpha^{(n+1)} = \frac{q^{(n+1)} - q^{(n)}}{\Delta t}$. What is the value of μ that is used in Crank-Nicolson method?

- (a) 1/4
- (b) 1/3
- (c) 1/2
- (d) 1

No, the answer is incorrect. Score: 0
Accepted Answers: (c) 1/2 2 points

10) After time discretization is used and the unknown α_{n+1} is written in terms of the known at t_n , one gets $[\bar{K}]\{\alpha^{(n+1)}\} = [\bar{K}]\{\alpha^{(n)}\} + \{F\}$; here the modified stiffness matrix $[\bar{K}]$ in the left hand side is obtained from the original matrix $[K]$ and $[M]$ as

- (a) $[\bar{K}] = [M] - \Delta t_n \mu [K]$
- (b) $[\bar{K}] = [K] + \Delta t_n \mu [M]$
- (c) $[\bar{K}] = [K] - \Delta t_n \mu [M]$
- (d) $[\bar{K}] = [M] + \Delta t_n \mu [K]$

No, the answer is incorrect. Score: 0
Accepted Answers: (d) $[\bar{K}] = [M] + \Delta t_n \mu [K]$ 2 points

11) After time discretization is used and the unknown α_{n+1} is written in terms of the known at t_n , one gets $[\bar{K}]\{\alpha^{(n+1)}\} = [\bar{K}]\{\alpha^{(n)}\} + \{F\}$; here the modified stiffness matrix $[\bar{K}]$ in the right hand side is obtained from the original matrix $[K]$ and $[M]$ as

- (a) $[\bar{K}] = [M] + \Delta t_n (1 - \mu)[K]$
- (b) $[\bar{K}] = [M] - \Delta t_n (1 + \mu)[K]$
- (c) $[\bar{K}] = [M] - \Delta t_n (1 - \mu)[K]$
- (d) $[\bar{K}] = [M] + \Delta t_n \mu [K]$

No, the answer is incorrect. Score: 0
Accepted Answers: (c) $[\bar{K}] = [M] - \Delta t_n (1 - \mu)[K]$ 2 points

12) After time discretization is used and the unknown α_{n+1} is written in terms of the known at t_n , one gets $[\bar{K}]\{\alpha^{(n+1)}\} = [\bar{K}]\{\alpha^{(n)}\} + \{F\}$; here the modified force vector $\{F\}$ in the right hand side is obtained from the original force vector $\{F\}$ as

- (a) $\{F\} = (1 - \mu)\{F^{(n)}\} + \mu\{F^{(n+1)}\}$
- (b) $\{F\} = (1 - \mu)\{F^{(n)}\} - \mu\{F^{(n+1)}\}$
- (c) $\{F\} = \Delta t_n [(1 - \mu)\{F^{(n)}\} + \mu\{F^{(n+1)}\}]$
- (d) $\{F\} = \Delta t_n [(1 - \mu)\{F^{(n)}\} - \mu\{F^{(n+1)}\}]$

No, the answer is incorrect. Score: 0
Accepted Answers: (c) $\{F\} = \Delta t_n [(1 - \mu)\{F^{(n)}\} + \mu\{F^{(n+1)}\}]$ 2 points

13) If the solution is represented in terms of the Lagrange basis functions $\phi_k(x)$ for degree $p=2$ as, $u = \sum_1^3 \alpha_k(t)\phi_k(x)$, then the Dirichlet boundary condition is represented by

- (a) $\alpha_1(t) = \alpha_2(t) = 0$
- (b) $\alpha_1(t) = 0$
- (c) $\alpha_2(t) = \alpha_3(t) = 0$
- (d) $\alpha_1(t) - \alpha_2(t) = 0$

No, the answer is incorrect. Score: 0
Accepted Answers: (b) $\alpha_1(t) = \alpha_2(t) = 0$ 2 points

14) Similarly, the initial condition of the problem is represented by,

- (a) $\alpha_1(0) = \alpha_2(0)$
- (b) $\alpha_2(0) = \alpha_3(0) = 0$
- (c) $\alpha_1(0) = \alpha_2(0) = 0$
- (d) $\alpha_1(0) = \alpha_2(0) = \alpha_3(0) = 0$

No, the answer is incorrect. Score: 0
Accepted Answers: (d) $\alpha_1(0) = \alpha_2(0) = \alpha_3(0) = 0$ 2 points

15) Calculate the value of \bar{K}_{22} (modified term in the left hand side) for $\Delta t_n = 0.5$.

Hint
No, the answer is incorrect. Score: 0
Accepted Answers: (Type: Range) 1.866,1.867 2 points

16) Calculate the value of \bar{K}_{22} (modified term in the right hand side) for $\Delta t_n = 0.5$.

Hint
No, the answer is incorrect. Score: 0
Accepted Answers: (Type: Range) -0.9,-0.7 2 points

17) Calculate the value of \bar{F}_2 (modified force term) for $\Delta t_n = 0.5$.

Hint
No, the answer is incorrect. Score: 0
Accepted Answers: (Type: Range) 0.33,0.34 2 points

18) Calculate the temperature at the mid point of the bar at $t = 0.5$ (assume $\Delta t_n = 0.1$).

Hint
No, the answer is incorrect. Score: 0
Accepted Answers: (Type: Range) 0.124,0.125 2 points

19) Calculate the temperature at the mid point of the bar at $t = 0.5$ (assume $\Delta t_n = 0.25$).

Hint
No, the answer is incorrect. Score: 0
Accepted Answers: (Type: Range) 0.123,0.124 2 points

20) Calculate the temperature at the mid point of the bar at $t = 0.5$ (assume $\Delta t_n = 0.5$).

Hint
No, the answer is incorrect. Score: 0
Accepted Answers: (Type: Range) 0.178,0.179 2 points