Unit 15 - Week 12: FEM formulation for 3D Elastic problem and challenges

Assignment 12

The due date for submitting this assignment has passed.
As per our records you have not submitted this assignment.

Due on 2020-12-09, 23:59 IST.

Consider an unsteady heat conduction problem in an insulated bar of length $l = 1$ as defined below. Assume one quadratic bar element for spatial discretization, and Crank-Nicolson scheme for time discretization to calculate the temperature at the mid point of the domain as time varies. Assume the heat generation term as $g = 1$. You may take help of MatLab for obtaining the solution.

$$\frac{\partial^2 u}{\partial x^2} + g = \frac{\partial u}{\partial t} \quad \text{for } 0 < x < 1$$
$$u(0,t) = u(1,t) = 0; \quad \text{(Boundary condition)}$$
$$u(x,0) = 0; \quad \text{(Initial condition)}$$

1) Choose the correct weak form. *2 points*

(a) $\int_0^l \frac{\partial u}{\partial x}\frac{\partial w}{\partial x}\,dx = \int_0^l gw\,dx + w\frac{\partial u}{\partial x}\Big|_0^l$

(b) $\int_0^l \frac{\partial u}{\partial t}\,w\,dx + \int_0^l \frac{\partial u}{\partial x}\frac{\partial w}{\partial x}\,dx = \int_0^l gw\,dx + w\frac{\partial u}{\partial x}\Big|_0^l$

(c) $\int_0^l \frac{\partial w}{\partial t}\,u\,dx + \int_0^l \frac{\partial u}{\partial x}\frac{\partial w}{\partial x}\,dx = \int_0^l gw\,dx + w\frac{\partial u}{\partial x}\Big|_0^l$

(d) $\int_0^l \frac{\partial u}{\partial t}\,w\,dx + \int_0^l \frac{\partial u}{\partial x}\frac{\partial w}{\partial x}\,dx = \int_0^l gw\,dx - w\frac{\partial u}{\partial x}\Big|_0^l$

No, the answer is incorrect.
Score: 0
Accepted Answers:
(b) $\int_0^l \frac{\partial u}{\partial t}\,w\,dx + \int_0^l \frac{\partial u}{\partial x}\frac{\partial w}{\partial x}\,dx = \int_0^l gw\,dx + w\frac{\partial u}{\partial x}\Big|_0^l$

2) Substituting $u = \sum \alpha_j\phi_j$ and $w = \phi_i$ in the weak form one obtains, $M_{ij}\dot{\alpha}_j + K_{ij}\alpha_j = F_i$, where $M_{ij} = \ldots$ *2 points*

(a) $M_{ij} = \int_0^l \phi_i\phi_{j,t}\,dx$

(b) $M_{ij} = \int_0^l \phi_i\phi_{j,x}\,dx$

(c) $M_{ij} = \int_0^l \phi_i\phi_j\,dx$

(d) $M_{ij} = \int_0^l \phi_{i,x}\phi_j\,dx$

No, the answer is incorrect.
Score: 0
Accepted Answers:
(c) $M_{ij} = \int_0^l \phi_i\phi_j\,dx$

3) Similarly, K_{ij} represents, *2 points*

(a) $K_{ij} = \int_0^l \phi_{i,x}\phi_{j,x}\,dx$

(b) $K_{ij} = \int_0^l \phi_i\phi_{j,x}\,dx$

(c) $K_{ij} = \int_0^l \phi_{i,x}\phi_j\,dx$

(d) $K_{ij} = \int_0^l \phi_i\phi_j\,dx$

No, the answer is incorrect.
Score: 0
Accepted Answers:
(a) $K_{ij} = \int_0^l \phi_{i,x}\phi_{j,x}\,dx$

4) Similarly, F_i represents, *2 points*

(a) $F_i = \int_0^l g\phi_i\,dx - \frac{\partial u}{\partial x}\phi_i\Big|_0^l$

(b) $F_i = \int_0^l g\phi_i\,dx + \frac{\partial u}{\partial x}\phi_i\Big|_0^l$

(c) $F_i = \int_0^l g\phi_i\,dx - \frac{\partial u}{\partial x}\phi_{i,x}\Big|_0^l$

(d) $F_i = \int_0^l g\phi_i\,dx + \frac{\partial u}{\partial x}\phi_{i,x}\Big|_0^l$

No, the answer is incorrect.
Score: 0
Accepted Answers:
(b) $F_i = \int_0^l g\phi_i\,dx + \frac{\partial u}{\partial x}\phi_i\Big|_0^l$

5) The definition of the shape function N_1 (Lagrange shape function for degree p=2) in the physical element is, *2 points*

(a) $N_1 = \frac{2}{l^2}(x - l/2)(x - l)$

(b) $N_1 = \frac{-4}{l^2}(x)(x - l)$

(c) $N_1 = \frac{-4}{l^2}(x)(x + l)$

(d) $N_1 = \frac{2}{l^2}(x)(x - l/2)$

No, the answer is incorrect.
Score: 0
Accepted Answers:
(a) $N_1 = \frac{2}{l^2}(x - l/2)(x - l)$

6) The definition of the shape function N_2 (Lagrange shape function for degree p=2) in the physical element is, *2 points*

(a) $N_2 = \frac{2}{l^2}(x - l/2)(x - l)$

(b) $N_2 = \frac{-4}{l^2}(x)(x - l)$

(c) $N_2 = \frac{-4}{l^2}(x)(x + l)$

(d) $N_2 = \frac{2}{l^2}(x)(x - l/2)$

No, the answer is incorrect.
Score: 0
Accepted Answers:
(b) $N_2 = \frac{-4}{l^2}(x)(x - l)$

7) The definition of the shape function N_3 (Lagrange shape function for degree p=2) in the physical element is, *2 points*

(a) $N_3 = \frac{2}{l^2}(x - l/2)(x - l)$

(b) $N_3 = \frac{-4}{l^2}(x)(x - l)$

(c) $N_3 = \frac{-4}{l^2}(x)(x + l)$

(d) $N_3 = \frac{2}{l^2}(x)(x - l/2)$

No, the answer is incorrect.
Score: 0
Accepted Answers:
(d) $N_3 = \frac{2}{l^2}(x)(x - l/2)$

8) The size of the so called stiffness matrix, [K], for this problem is, *2 points*

(a) 4×4

(b) 6×6

(c) 2×2

(d) 3 × 3

No, the answer is incorrect.
Score: 0
Accepted Answers:
(d) 3 × 3

9) During time discretization we use, $(1-\mu)\dot{\alpha}^{(n)} + \mu\dot{\alpha}^{(n+1)} = \frac{\alpha^{(n+1)} - \alpha^{(n)}}{\Delta t^n}$. What is the value of μ that is used in Crank-Nicolson method? *2 points*

(a) 1/4
(b) 1/3
(c) 1/2
(d) 1

No, the answer is incorrect.
Score: 0
Accepted Answers:
(c) 1/2

10) After time discretization is used and the unknown α_{n+1} is written in terms of the known at t_n, one gets $[\bar{K}]\{\alpha^{(n+1)}\} = [\bar{\bar{K}}]\{\alpha^{(n)}\} + \{\bar{F}\}$; here the modified stiffness matrix $[\bar{K}]$ in the left hand side is obtained from the original matrix [K] and [M] as *2 points*

(a) $[\bar{K}] = [M] - \Delta t_n \mu [K]$

(b) $[\bar{K}] = [K] + \Delta t_n \mu [M]$

(c) $[\bar{K}] = [K] - \Delta t_n \mu [M]$

(d) $[\bar{K}] = [M] + \Delta t_n \mu [K]$

No, the answer is incorrect.
Score: 0
Accepted Answers:
(d) $[\bar{K}] = [M] + \Delta t_n \mu [K]$

11) After time discretization is used and the unknown α_{n+1} is written in terms of the known at t_n, one gets $[\bar{K}]\{\alpha^{(n+1)}\} = [\bar{\bar{K}}]\{\alpha^{(n)}\} + \{\bar{F}\}$; here the modified stiffness matrix $[\bar{\bar{K}}]$ in the right hand side is obtained from the original matrix [K] and [M] as *2 points*

(a) $[\bar{\bar{K}}] = [M] + \Delta t_n(1-\mu)[K]$

(b) $[\bar{\bar{K}}] = [M] - \Delta t_n(1+\mu)[K]$

(c) $[\bar{\bar{K}}] = [M] - \Delta t_n(1-\mu)[K]$

(d) $[\bar{\bar{K}}] = [M] + \Delta t_n \mu [K]$

No, the answer is incorrect.
Score: 0
Accepted Answers:
(c) $[\bar{\bar{K}}] = [M] - \Delta t_n(1-\mu)[K]$

12) After time discretization is used and the unknown α_{n+1} is written in terms of the known at t_n, one gets $[\bar{K}]\{\alpha^{(n+1)}\} = [\bar{\bar{K}}]\{\alpha^{(n)}\} + \{\bar{F}\}$; here the modified force vector $\{\bar{F}\}$ in the right hand side is obtained from the original force vector $\{F\}$ as *2 points*

(a) $\{\bar{F}\} = (1-\mu)\{F^{(n)}\} + \mu\{F^{(n+1)}\}$

(b) $\{\bar{F}\} = (1-\mu)\{F^{(n)}\} - \mu\{F^{(n+1)}\}$

(c) $\{\bar{F}\} = \Delta t_n[(1-\mu)\{F^{(n)}\} + \mu\{F^{(n+1)}\}]$

(d) $\{\bar{F}\} = \Delta t_n[(1-\mu)\{F^{(n)}\} - \mu\{F^{(n+1)}\}]$

No, the answer is incorrect.
Score: 0
Accepted Answers:
(c) $\{\bar{F}\} = \Delta t_n[(1-\mu)\{F^{(n)}\} + \mu\{F^{(n+1)}\}]$

13) If the solution is represented in terms of the Lagrange basis functions $\phi_i(x)$ for degree p=2 as, $u = \sum_1^3 \alpha_i(t)\phi_i(x)$, then the Dirichlet boundary condition is represented by *2 points*

(a) $\alpha_1(t) = \alpha_3(t) = 0$

(b) $\alpha_1(t) = 0$

(c) $\alpha_2(t) = \alpha_3(t) = 0$

(d) $\alpha_1(t) - \alpha_3(t) = 0$

No, the answer is incorrect.
Score: 0
Accepted Answers:
(a) $\alpha_1(t) = \alpha_3(t) = 0$

14) Similarly, the initial condition of the problem is represented by, *2 points*

(a) $\alpha_1(0) = \alpha_2(0)$

(b) $\alpha_2(0) = \alpha_3(0) = 0$

(c) $\alpha_1(0) = \alpha_3(0) = 0$

(d) $\alpha_1(0) = \alpha_2(0) = \alpha_3(0) = 0$

No, the answer is incorrect.
Score: 0
Accepted Answers:
(d) $\alpha_1(0) = \alpha_2(0) = \alpha_3(0) = 0$

15) Calculate the value of $\bar{K}_{22}$ (modified term in the left hand side) for $\Delta t_n = 0.5$.

Hint

No, the answer is incorrect.
Score: 0
Accepted Answers:
(Type: Range) 1.866,1.867

2 points

16) Calculate the value of $\bar{\bar{K}}_{22}$ (modified term in the right hand side) for $\Delta t_n = 0.5$.

Hint

No, the answer is incorrect.
Score: 0
Accepted Answers:
(Type: Range) -0.9,-0.7

2 points

17) Calculate the value of $\bar{F}_2$ (modified force term) for $\Delta t_n = 0.5$.

Hint

No, the answer is incorrect.
Score: 0
Accepted Answers:
(Type: Range) 0.33,0.34

2 points

18) Calculate the temperature at the mid point of the bar at t = 0.5 (assume $\Delta t_n = 0.1$).

Hint

No, the answer is incorrect.
Score: 0
Accepted Answers:
(Type: Range) 0.124,0.125

2 points

19) Calculate the temperature at the mid point of the bar at t = 0.5 (assume $\Delta t_n = 0.25$).

Hint

No, the answer is incorrect.
Score: 0
Accepted Answers:
(Type: Range) 0.123,0.124

2 points

20) Calculate the temperature at the mid point of the bar at t = 0.5 (assume $\Delta t_n = 0.5$).

Hint

No, the answer is incorrect.
Score: 0
Accepted Answers:
(Type: Range) 0.178,0.179

2 points