

Unit 14 - Week 11: Eigen value problems and Transient problem in 1D & 2D Scalar Valued Problems

Course outline
How does an NPTEL online course work?
MATLAB
Week 0: Prerequisite
Week 1: Variational Calculus and Minimization Problem
Week 2: One dimensional Finite Element Analysis
Week 3: Structural Elements in One Dimensional FEM
Week 4: Structural Elements in One Dimensional FEM
Week 5: Structural Elements in One Dimensional FEM, and Generalized One Dimensional Finite Element Code in Computer Programming
Week 6: Brief Background of Tensor Calculus
Week 7: Two dimensional Scalar field problems
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Week 9: Two dimensional Scalar and Vector field problems
Week 10: Two dimensional Vector field and Eigen value problems
Week 11: Eigen value problems and Transient problem in 1D & 2D Scalar Valued Problems
<input type="radio"/> Lec 32: Solving eigenvalue problem of membrane, writing FEM code in MATLAB
<input type="radio"/> Lec 33: Solving transient problems (parabolic type)
<input type="radio"/> Lec 34: Solving transient problems (hyperbolic type)
<input type="radio"/> Quiz : Assignment 11
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Week 12: FEM formulation for 3D Elastic problem and challenges
Live session: Dr. Atanu Banerjee, Date : 16/12/2020 Time : 3:15:00 PM

Assignment 11

The due date for submitting this assignment has passed. As per our records you have not submitted this assignment.

Due on 2020-12-02, 23:59 IST.

We need to find out the critical load for the column AB, shown in Fig 1, to buckle, using finite element method. The column is pinned at A and B. You can start with the differential equation shown in Equation 1, where $y(x)$ refers the transverse displacement of the column. Model the whole column with one beam element. The Hermite cubic shape functions are expressed in Equation 2. Assume the column to have uniform flexural rigidity EI and is of length L and answer the following questions.

$$\frac{d^4}{dx^4} \left(EI \frac{d^2 y}{dx^2} \right) + P_0 \frac{d^2 y}{dx^2} = 0, \quad \Omega \in (0, L) \quad \dots\dots\dots(1)$$

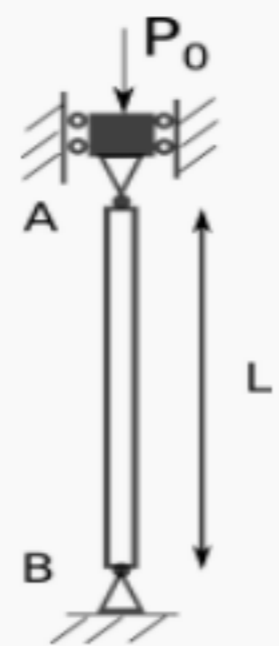


Figure (1)

$$\phi_1 = \{1 - 3(x/L)^2 + 2(x/L)^3\}; \quad \phi_2 = x\{1 - (x/L)\}^2, \quad \dots\dots\dots(2)$$

$$\phi_3 = \{3(x/L)^2 - 2(x/L)^3\}; \quad \phi_4 = x\{(x/L)^2 - (x/L)\}.$$

1) After taking care of Dirichlet Boundary conditions, the weak form (the usual weight function is represented by w) of the problem yields, (Here $(\cdot)' = \frac{d}{dx}(\cdot)$, $(\cdot)'' = \frac{d^2}{dx^2}(\cdot)$ and so on) 2 points

- $\int_0^L EI y'' w'' dx = \int_0^L P_0 y' w dx$
- $\int_0^L EI y'' w'' dx = \int_0^L P_0 y' w' dx$
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- $\int_0^L EI y'' w'' dx = \int_0^L P_0 y w'' dx$

No, the answer is incorrect.

Score: 0

Accepted Answers:

$$\int_0^L EI y'' w'' dx = \int_0^L P_0 y' w' dx$$

2) The boundary conditions for the problem can be represented as, (Here $(\cdot)' = \frac{d}{dx}(\cdot)$, $(\cdot)'' = \frac{d^2}{dx^2}(\cdot)$ and so on) 2 points

- $y(x=0) = y(x=L) = 0$ and $y'(x=0) = y'(x=L) = 0$
- $y(x=0) = y'(x=0) = 0$ and $y''(x=0) = y''(x=L) = 0$
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No, the answer is incorrect.

Score: 0

Accepted Answers:

$$y(x=0) = y(x=L) = 0 \text{ and } y''(x=0) = y''(x=L) = 0$$

3) Calculate the stiffness entry K_{22} . 2 points

- $\frac{4EI}{L}$
- $\frac{4EI}{L^2}$
- $\frac{4EI}{L^3}$
- $\frac{2EI}{L}$
- $\frac{2EI}{L^2}$

No, the answer is incorrect.

Score: 0

Accepted Answers:

$$\frac{4EI}{L}$$

4) Calculate the stiffness entry K_{24} . 2 points

- $\frac{4EI}{L}$
- $\frac{4EI}{L^2}$
- $\frac{2EI}{L}$
- $\frac{2EI}{L^2}$

No, the answer is incorrect.

Score: 0

Accepted Answers:

$$\frac{2EI}{L}$$

5) Calculate the stiffness entry K_{44} . 2 points

- $\frac{4EI}{L}$
- $\frac{4EI}{L^2}$
- $\frac{2EI}{L}$
- $\frac{2EI}{L^2}$

No, the answer is incorrect.

Score: 0

Accepted Answers:

$$\frac{4EI}{L}$$

6) Calculate M_{23} , where $M_{ij} = \int_0^L \phi_{i,x} \phi_{j,x} dx$ and $\phi_{1,x} = \phi_1' = \frac{6x}{L}$. 2 points

- $\frac{4L}{15}$
- $\frac{2L}{15}$
- $\frac{1}{10}$
- $\frac{L}{30}$

No, the answer is incorrect.

Score: 0

Accepted Answers:

$$\frac{2L}{15}$$

7) Calculate M_{34} , where $M_{ij} = \int_0^L \phi_{i,x} \phi_{j,x} dx$ and $\phi_{1,x} = \phi_1' = \frac{6x}{L}$. 2 points

- $\frac{4L}{15}$
- $-\frac{1}{10}$
- $-\frac{L}{30}$
- $\frac{L}{30}$

No, the answer is incorrect.

Score: 0

Accepted Answers:

$$-\frac{L}{30}$$

8) Calculate M_{44} , where $M_{ij} = \int_0^L \phi_{i,x} \phi_{j,x} dx$ and $\phi_{1,x} = \phi_1' = \frac{6x}{L}$. 2 points

- $\frac{4L}{15}$
- $-\frac{1}{30}$
- $\frac{L}{30}$
- $\frac{2L}{15}$

No, the answer is incorrect.

Score: 0

Accepted Answers:

$$\frac{2L}{15}$$

9) After enforcing the Dirichlet boundary conditions, solve the eigen value problem (may use MATLAB) and calculate the smallest value of P_c . 2 points

- $\frac{9EI}{L^2}$
- $\frac{12EI}{L^2}$
- $\frac{8EI}{L^2}$
- $\frac{10EI}{L^2}$

No, the answer is incorrect.

Score: 0

Accepted Answers:

$$\frac{12EI}{L^2}$$

10) What is the critical load for this column as obtained using the analytical method? 2 points

- $\frac{\pi^2 EI}{4L^2}$
- $\frac{\pi^2 EI}{L^2}$
- $\frac{4\pi^2 EI}{L^2}$
- $\frac{\pi^2 EI}{L}$

No, the answer is incorrect.

Score: 0

Accepted Answers:

$$\frac{\pi^2 EI}{L^2}$$

11) Discretize the domain with two equal beam elements and obtain the smallest critical load. (You may take help of MATLAB for solving this part of the problem.) 10 points

- $9.8696 \frac{EI}{L^2}$
- $9.9023 \frac{EI}{L^2}$
- $9.8038 \frac{EI}{L^2}$
- $9.9438 \frac{EI}{L^2}$

No, the answer is incorrect.

Score: 0

Accepted Answers:

$$9.9438 \frac{EI}{L^2}$$