

## Unit 4 - Week 1: Variational Calculus and Minimization Problem

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Live session: Dr. Atanu Banerjee, Date : 16/12/2020 Time : 3:15:00 PM

### Assignment 1

The due date for submitting this assignment has passed. As per our records you have not submitted this assignment. **Due on 2020-09-30, 23:59 IST.**

Answer the question from "a" to "d" (i.e 1 to 13) with the help of below statement:  
Consider the following differential equation on the domain [1,3]:

$$x^2 \frac{d^2 y}{dx^2} + 6x \frac{dy}{dx} + 6 \frac{d^2 y}{dx^2} - 10x = 0.$$

The boundary conditions are  $y = y' = 0$  at  $x = 1$  and  $x = 3$

(a) We can derive the variational statement as

$$J[y] = \int_1^3 \left[ a^2 \left( \frac{dy}{dx} \right)^2 + \left( \frac{d^2 y}{dx^2} \right)^2 \right] dx = \int_1^3 b(x) dx$$

1) What is the value of  $a$ ? 2 points

A. 1  
 B. 2  
 C. 3  
 D. 0

No, the answer is incorrect. Score: 0

Accepted Answers: C, 3

2) What are the values of  $l$  and  $m$ ? 2 points

A. 2 & 2  
 B. 2 & 1  
 C. 1 & 2  
 D. 1 & 1

No, the answer is incorrect. Score: 0

Accepted Answers: D, 1 & 1

3) What is the value of  $h$ ? 2 points

A. 5  
 B. 10  
 C. 2  
 D. 4

No, the answer is incorrect. Score: 0

Accepted Answers: B, 10

4) What is the condition to have minimization problem? 2 points

A.  $m \neq l$   
 B.  $m > l$   
 C.  $m < l$   
 D.  $m = l$

No, the answer is incorrect. Score: 0

Accepted Answers: D, m = l

(b) Minimization statement will be of the following form -  
Find  $y$  such that,

$$\Pi(y) = \text{Min}_y \Pi(y) = \text{Min}_y \frac{1}{2} \int_1^3 \left[ a^2 \left( \frac{dy}{dx} \right)^2 - p(x) \right] dx$$

5) What is the value of  $c$ ? 1 point

A. 2  
 B. 4  
 C. 1  
 D. 3

No, the answer is incorrect. Score: 0

Accepted Answers: A, 2

6) What is the value of  $p$ ? 1 point

A. 10  
 B. 5  
 C. 20  
 D. 4

No, the answer is incorrect. Score: 0

Accepted Answers: C, 20

(c) In Rayleigh Ritz method, we can approximate the solution as,  
 $y = C_1 \phi_1(x) + \phi_2(x)$

where  $\phi_1(x)$  is taken as  $(x - 1)(x - 3)^m$

7) What is the expression for  $\phi_2(x)$ ? 1 point

A. 1  
 B. 3  
 C. 0  
 D. 4

No, the answer is incorrect. Score: 0

Accepted Answers: C, 0

8) Find the minimum required value of  $l$  and  $m$ ? 2 points

A. 2 & 2  
 B. 1 & 1  
 C. 3 & 2  
 D. 1 & 2

No, the answer is incorrect. Score: 0

Accepted Answers: A, 2 & 2

(d) Solve the problem using Rayleigh Ritz method with above values of  $l$  and  $m$ .  
Eventually we will have an equation of the form,

$$\int_1^3 x^2 [px^2 + qx + r]^2 dx = 10 \int_1^3 [x^3 - 8x^4 + ax^2 - 24x^3 + 9x] dx$$

9) What is the values of  $p$ ? 1 point

A. 10  
 B. 15  
 C. 12  
 D. 4

No, the answer is incorrect. Score: 0

Accepted Answers: C, 12

10) What is the values of  $q$ ? 1 point

A. -48  
 B. 48  
 C. 15  
 D. -15

No, the answer is incorrect. Score: 0

Accepted Answers: A, -48

11) What is the values of  $r$ ? 1 point

A. 10  
 B. 12  
 C. 44  
 D. 36

No, the answer is incorrect. Score: 0

Accepted Answers: C, 44

12) What is the values of  $a$ ? 1 point

A. 12  
 B. 22  
 C. 32  
 D. 42

No, the answer is incorrect. Score: 0

Accepted Answers: B, 22

13) Find the value of  $C_1$ ?

**Hint**

No, the answer is incorrect. Score: 0

Accepted Answers: (Type: Range) 0/2, 0/6

Answer the question from "a" and "b" (i.e 14 to 27) with the help of below statement:  
A steel rod of diameter  $d = 2$  cm, length  $L = 25$  cm, and thermal conductivity  $k = 50W/(m^{\circ}C)$  is exposed to ambient air  $T_{\infty} = 20^{\circ}C$ , with a heat transfer coefficient  $\beta = 64W/(m^2 \cdot ^{\circ}C)$ . Given that the left end of the rod is maintained at a temperature of  $T_0 = 120^{\circ}C$ . The equation governing the problem is given by

$$-\frac{d^2 \theta}{dx^2} + c\theta = 0 \quad \text{for } 0 < x < 25 \text{ cm,}$$

where  $\theta = T - T_{\infty}$ , and  $c$  is given by  $c = \frac{dP}{dA}$   
where  $P$  is the perimeter and  $A$  is the cross-sectional area of the rod. The boundary conditions are

$$\theta(0) = T(0) - T_{\infty} = 100, \left( k \frac{d\theta}{dx} + \beta \theta \right) = 0 \text{ at } x = L.$$

(a) We can derive the variational statement as

$$J[\theta] = \int_0^L \left[ \left( \frac{d\theta}{dx} \right)^2 + c\theta \right] dx = a(\theta)_{x=0}$$

14) What is the value of  $a$ ? 2 points

A.  $-\frac{64}{25}$   
 B.  $\frac{64}{25}$   
 C.  $-\frac{90}{21}$   
 D.  $\frac{90}{14}$

No, the answer is incorrect. Score: 0

Accepted Answers: A,  $-\frac{64}{25}$

15) What are the values of  $l$  and  $m$ ? 2 points

A. 2 & 2  
 B. 2 & 1  
 C. 1 & 2  
 D. 1 & 1

No, the answer is incorrect. Score: 0

Accepted Answers: D, 1 & 1

16) What is the condition to have minimization problem? 2 points

A.  $m \neq l$   
 B.  $m > l$   
 C.  $m < l$   
 D.  $m = l$

No, the answer is incorrect. Score: 0

Accepted Answers: D, m = l

17) What is the value of  $c$ ? 2 points

A. 256m<sup>2</sup>  
 B. 512m<sup>2</sup>  
 C. 768m<sup>2</sup>  
 D. 1038m<sup>2</sup>

No, the answer is incorrect. Score: 0

Accepted Answers: A, 256m<sup>2</sup>

(b) We can approximate the solution as,

$$\theta = C_1 \phi_1(x) + C_2 \phi_2(x) + \phi_3(x) \quad \text{and} \quad v = \phi_i(x) \quad \text{where } i = 1, 2$$

18) What is the suitable expressions for  $\phi_1(x)$ ,  $\phi_2(x)$  and  $\phi_3(x)$ ? 2 points

A.  $\phi_1(x) = \frac{1}{x}$   
 $\phi_2(x) = \frac{1}{x^2}$   
 $\phi_3(x) = 100$

B.  $\phi_1(x) = x^2$   
 $\phi_2(x) = x + x^2$   
 $\phi_3(x) = 50$

C.  $\phi_1(x) = (x + 1)^2$   
 $\phi_2(x) = x^2 + 1$   
 $\phi_3(x) = 100$

D.  $\phi_1(x) = x$   
 $\phi_2(x) = x^2$   
 $\phi_3(x) = 100$

No, the answer is incorrect. Score: 0

Accepted Answers: D,  $\phi_1(x) = x$   
 $\phi_2(x) = x^2$   
 $\phi_3(x) = 100$

Answer the questions from 19 to 21 with the help of below statement:

For  $v = \phi_1(x)$ , variational statement will be reduced to  $a_1 C_1 + a_2 C_2 + a_3 = 0$  then,  
(Use expression of  $\phi_1(x)$  as selected in previous question Q. No: 18)

19) The value of  $a_1$  is?

**Hint**

No, the answer is incorrect. Score: 0

Accepted Answers: (Type: Range) 1/3, 1/7

20) The value of  $a_2$  is?

**Hint**

No, the answer is incorrect. Score: 0

Accepted Answers: (Type: Range) 2/3, 3/7

21) The value of  $a_3$  is?

**Hint**

No, the answer is incorrect. Score: 0

Accepted Answers: (Type: Range) 6/25, 6/40

Answer the question from 22 to 24 with the help of below statement:  
For  $v = \phi_2(x)$ , variational statement will be reduced to  $b_1 C_1 + b_2 C_2 + b_3 = 0$  then, (Use expression of  $\phi_2(x)$  as selected in previous question Q. No: 18)

22) The value of  $b_1$  is?

**Hint**

No, the answer is incorrect. Score: 0

Accepted Answers: (Type: Range) 2/3, 3/5

23) The value of  $b_2$  is?

**Hint**

No, the answer is incorrect. Score: 0

Accepted Answers: (Type: Range) 0/2, 0/8

24) The value of  $b_3$  is?

**Hint**

No, the answer is incorrect. Score: 0

Accepted Answers: (Type: Range) 1/40, 3/14, 8

25) What is the value of  $C_1$ ?

**Hint**

No, the answer is incorrect. Score: 0

Accepted Answers: (Type: Range) -1/40, -1/20

26) What is the value of  $C_2$ ?

**Hint**

No, the answer is incorrect. Score: 0

Accepted Answers: (Type: Range) 2/65, 2/70

27) What will be the temperature at the centre of the rod?

**Hint**

No, the answer is incorrect. Score: 0

Accepted Answers: (Type: Range) 3/1, 3/4

2 points