Assignment 8

The date for submitting this assignment has been changed. Your final grade on this course will be determined by your performance on the final exam. If you have not already completed this assignment, you are encouraged to do so now.

1. Question 1. Let \( a = \{a_1, a_2, a_3, a_4, a_5\} \in \{0, 1\}^5 \) and \( b = \{b_1, b_2, b_3, b_4, b_5\} \in \{0, 1\}^5 \). Let \( a^* = (1, 2) \). Which of the following are statements that are true or false?

   - True
   - False

2. Question 2. Let \( x = \{x_1, x_2, x_3\} \in \{0, 1\}^3 \) and \( y = \{y_1, y_2, y_3\} \in \{0, 1\}^3 \). Suppose \( T \neq \emptyset \). The largest value of \( x^* = (1, 2) \) is a solution if and only if:

   - True
   - False

3. Question 3. Consider the problem:

   \[
   \begin{align*}
   \text{minimize} & \quad f(x) \\
   \text{subject to} & \quad x_1 + x_2 = 2, \\
   & \quad x_1 + x_2 = 3, \\
   & \quad x_1, x_2 \geq 0.
   \end{align*}
   \]

   How many solutions are there or does the problem have no solution?

4. Question 4. Consider the problem:

   \[
   \begin{align*}
   \text{minimize} & \quad f(x) \\
   \text{subject to} & \quad x_1 + x_2 = 2, \\
   & \quad x_1 + x_2 = 3, \\
   & \quad x_1, x_2 \geq 0.
   \end{align*}
   \]

5. Question 5. Consider the problem:

   \[
   \begin{align*}
   \text{minimize} & \quad f(x) \\
   \text{subject to} & \quad x_1 + x_2 = 2, \\
   & \quad x_1 + x_2 = 3, \\
   & \quad x_1, x_2 \geq 0.
   \end{align*}
   \]

   Note: TRUE or FALSE? Is \( x^* \) satisfies \( f(x) = 1 \)?

6. Question 6. Consider the problem:

   \[
   \begin{align*}
   \text{minimize} & \quad f(x) \\
   \text{subject to} & \quad x_1 + x_2 = 2, \\
   & \quad x_1 + x_2 = 3, \\
   & \quad x_1, x_2 \geq 0.
   \end{align*}
   \]

   Note: TRUE or FALSE? \( x^* \) satisfies \( f(x) = 2 \)?

7. Question 7. Consider the problem:

   \[
   \begin{align*}
   \text{minimize} & \quad f(x) \\
   \text{subject to} & \quad x_1 + x_2 = 2, \\
   & \quad x_1 + x_2 = 3, \\
   & \quad x_1, x_2 \geq 0.
   \end{align*}
   \]

   Note: TRUE or FALSE? \( x^* \) satisfies \( f(x) = 3 \)?

8. Question 8. Consider the problem:

   \[
   \begin{align*}
   \text{minimize} & \quad f(x) \\
   \text{subject to} & \quad x_1 + x_2 = 2, \\
   & \quad x_1 + x_2 = 3, \\
   & \quad x_1, x_2 \geq 0.
   \end{align*}
   \]

   Note: TRUE or FALSE? \( x^* \) satisfies \( f(x) = 4 \)?

9. Question 9. Let \( a^* = (1, 2) \) be the solution of a convex function \( f : \mathbb{R}^n \to \mathbb{R} \). If \( T \neq \emptyset \), then \( T = \{a^*\} \).

   - True
   - False

10. Question 10. Let \( X \) be a convex subset of \( \mathbb{R}^n \). Let \( a^* \) be a solution of \( f \). If \( f \) is convex, then \( a^* \) is a global minimum.

   - True
   - False

11. Question 11. Let \( f : \mathbb{R}^n \to \mathbb{R} \) be defined as \( f(x_1, x_2) = x_1 + x_2 \). Consider the problem:

   \[
   \begin{align*}
   \text{minimize} & \quad f(x) \\
   \text{subject to} & \quad x_1 + x_2 = 2, \\
   & \quad x_1, x_2 \geq 0.
   \end{align*}
   \]

   Let \( x^* = (a, b) \) be the optimal solution. What is \( f(x^*) \)?

   - True
   - False