Week 6 Assessment

Due on 2021-06-05, 20:59 HRT.

1. Let \( \mathbf{u} \) be the fundamental solution of the heat equation. Then,

\[
\frac{\partial \mathbf{u}}{\partial t}(x,0) = \mathbf{u}(x,0)
\]

2. Let \( \mathbf{u}(x,t) \) be the fundamental solution of the heat equation. Then,

\[
\frac{\partial^2 \mathbf{u}}{\partial x^2}(x,0) = 0
\]

3. Let \( \mathbf{u}(x,t) \) be the fundamental solution of the heat equation. Then,

\[
\frac{\partial \mathbf{u}}{\partial t}(x,0) = \mathbf{u}(x,0)
\]

4. Let \( \mathbf{u}(x,t) \) be the fundamental solution of the heat equation. Then,

\[
\frac{\partial \mathbf{u}}{\partial t}(x,0) = \mathbf{u}(x,0)
\]

5. Which of the following properties holds true for the heat equation? *

- It is linear.
- It is nonlinear.
- The speed of propagation is infinite.
- The speed of propagation is finite.

6. Suppose the function is a continuous function \( f \). Find the partial derivatives \( u_{xx} \) and \( u_{x} \) at \( x = 0 \) and \( t = 0 \) and are continuous. Then,

\[
\frac{\partial^2 u}{\partial x^2}(x,0) = 0
\]

7. Let \( u \) be the smooth solution of the PDE.

\[
\begin{align*}
\frac{\partial u}{\partial t} + \nabla \cdot (\mathbf{u} \nabla u) &= 0 \\
\Delta u + \nabla \cdot (\mathbf{v} \nabla u) &= 0
\end{align*}
\]

8. Let \( u \) be the smooth solution of the following initial-boundary value problem:

\[
\begin{align*}
\frac{\partial u}{\partial t} + \nabla \cdot (\mathbf{u} \nabla u) &= 0 \\
\Delta u + \nabla \cdot (\mathbf{v} \nabla u) &= 0
\end{align*}
\]

9. Consider the function \( f(x) \). Then,

\[
\int_{a}^{b} f(x) \, dx = 0
\]

10. Consider the function \( f(x) \). Then,

\[
\int_{a}^{b} f(x) \, dx = 0
\]