Let \( X = \mathbb{R} \), \( \mathcal{F} = \{ A \subseteq \mathbb{R} \mid A \text{ is countable or } A^c \text{ is countable} \} \). Let \( \mu(A) = \begin{cases} 1 & \text{if } A^c \text{ is countable} \\ 0 & \text{if } A \text{ is countable} \end{cases} \)

Let \( f : (X, \mathcal{F}, \mu) \rightarrow \mathbb{R} \) be measurable. Which of the following are always true?

- \( f \) is constant ae.\( (\mu) \)
- \( f \) is a non-constant continuous function
- \( f \) is a non constant polynomial
- \( f(x) = 0 \ \forall \ x \in \mathbb{R} \)

No, the answer is incorrect.

Score: 0

Accepted Answers:
- \( f \) is constant ae.\( (\mu) \)

2) Let \( (X, \mathcal{F}, \mu) \) be a measure space and let \( E \) be a proper subset of \( X \), \( E \in \mathcal{F} \) and \( 0 < \mu(E) < \mu(X) \).

\[
f_n = \begin{cases} \chi_E & \text{if } n \text{ is odd} \\ 1 - \chi_E & \text{if } n \text{ is even} \end{cases}
\]

Due on 2020-02-12, 23:59 IST. As per our records you have not submitted this assignment.
Dominated convergence theorem (unit? unit=24&lesson=28)

Sets of measure zero and completion (unit? unit=24&lesson=29)

Quiz : Week 2 Assessment (assessment? name=94)

Outer measure

Lebesgue measure and its properties

Lebesgue measure and positive Borel measures on locally compact spaces

Lebesgue measure and invariance properties

L^p spaces and completeness

Product spaces and Fubini's theorem

Applications of Fubini's theorem and complex measures

Complex measures and Radon-Nikodym theorem

Radon-Nikodym theorem and applications

Riesz representation theorem and Lebesgue

\[ \int_X \liminf f_n \, d\mu < \liminf \int_X f_n \, d\mu \]

\[ \int_X \limsup f_n \, d\mu = \limsup \int_X f_n \, d\mu \]

\[ \int_X \limsup f_n \, d\mu < \limsup \int_X f_n \, d\mu \]

\[ \int_X \limsup f_n \, d\mu = \limsup \int_X f_n \, d\mu \]

No, the answer is incorrect.

Score: 0

Accepted Answers:

3) Consider the space \( \mathbb{N} \) with power set sigma algebra and counting measure \( \mu \). Let \( f : \mathbb{N} \to \mathbb{R} \) be measurable and zero a.e(\( \mu \)). Which of the following are always true?  

- \( f(n) = 0 \ \forall n \in \mathbb{N} \)
- \( f(1) \neq 0, f(n) = 0 \ \forall n > 1 \)
- \( f(n) = 0 \) except for finitely many \( n \in \mathbb{N} \)
- \( f(n) = 0 \) only when \( n \) is a prime number

No, the answer is incorrect.

Score: 0

Accepted Answers:

4) Let \( X \) be a non empty set and \( A \subset X \) be a proper subset. Consider the sigma algebra \( \mathcal{F} = \{ \phi, X, A, A^c \} \). Let \( f : (X, \mathcal{F}) \to \mathbb{R} \) be measurable. Which of the following are always true?  

- \( f = a \chi_A + b \chi_{A^c} \) for some \( a, b \in \mathbb{R} \)
- \( f = a \chi_A \) for some \( a \in \mathbb{R} \)
- \( f = b \chi_{A^c} \) for some \( b \in \mathbb{R} \)
- \( f \equiv 0 \)

No, the answer is incorrect.

Score: 0

Accepted Answers:

5) Let \( (X, \mathcal{F}, \mu) \) be a measure space. Let \( A_n \in \mathcal{F} \) be such that \( A_1 \subset A_2 \subset A_3 \subset \cdots \) and \( \bigcup_{n=1}^{\infty} A_n = X \). Let \( f : (X, \mathcal{F}, \mu) \to \mathbb{R} \) be a measurable function and \( f(x) \geq 0 \) a.e(\( \mu \)). Which of the following are always true?  

- \( f|_{A_n} \uparrow f \) ae
- \( \int_{A_n} f \, d\mu \uparrow \int_X f \, d\mu \)

No, the answer is incorrect.

Score: 0

Accepted Answers:
6) Let \((X, \mathcal{F}, \mu)\) be a measure space and \(\mu\) be a probability measure, that is \(\mu(X) = 1\). Let \(\{A_n\}\) be a sequence in \(\mathcal{F}\). Which of the following are true? \(1\) point

- \(\mu(\limsup A_n) \geq \limsup \mu(A_n)\)
- \(\mu(\limsup A_n) \leq \limsup \mu(A_n)\)
- \(\mu(\liminf A_n) \geq \liminf \mu(A_n)\)
- \(\mu(\liminf A_n) \leq \liminf \mu(A_n)\)

No, the answer is incorrect. Score: 0
Accepted Answers:
- \(\mu(\limsup A_n) \geq \limsup \mu(A_n)\)
- \(\mu(\liminf A_n) \leq \liminf \mu(A_n)\)

7) Let \((X, \mathcal{F}, \mu)\) be a measure space. Let \(f_n : X \to \mathbb{R}\) be measurable, \(A = \{x \in X \mid \lim f_n(x) \text{ exists}\}\). Then, \(1\) point

- \(A \in \mathcal{F}\)
- \(A = \emptyset\)
- \(A = X\)
- \(A^c \in \mathcal{F}\)

No, the answer is incorrect. Score: 0
Accepted Answers:
- \(A \in \mathcal{F}\)
- \(A^c \in \mathcal{F}\)

8) Let \((X, \mathcal{F}, \mu)\) be a measure space and \(A_n \in \mathcal{F}\). Suppose \(\mu(X) = 1, \sum \mu(A_n) < \infty\). \(1\) point
Then which of the following are true?

- \(\mu(\limsup A_n) = 0\)
- \(\mu(\liminf A_n) = 0\)
- \(\mu(\limsup A_n) = 1\)
- \(\mu(\liminf A_n) = 1\)
No, the answer is incorrect.
Score: 0
Accepted Answers:
$\mu(\limsup A_n) = 0$
$\mu(\liminf A_n) = 0$

9) $(X, \mathcal{F}, \mu)$ be a probability measure space. Suppose $f_n : X \to \mathbb{R}$ are measurable and $|f_n| \leq 1$ ae($\mu$) $\forall n$. Suppose $f_n \to 1$ ae($\mu$). Then,

- $\int_X f_n d\mu \to 1$
- $\int_X f_n d\mu \to 0$
- $\int_X f_n d\mu \to \infty$
- $\int_X f_n d\mu$ does not converge

No, the answer is incorrect.
Score: 0
Accepted Answers:
$\int_X f_n d\mu \to 1$

10) Let $(X, \mathcal{F}, \mu)$ be a measure space and $A_n \in \mathcal{F}$ such that $\mu(A_n) = 0$ $\forall n$. Which of the following are true?

- $\mu(\bigcup_{n=1}^{\infty} A_n) = 0$
- $\mu(\bigcup_{n=1}^{\infty} A_n) = 1$
- $\mu(\bigcup_{n=1}^{\infty} A_n) = \infty$
- $\mu(\bigcup_{n=1}^{\infty} A_n) > 0$

No, the answer is incorrect.
Score: 0
Accepted Answers:
$\mu(\bigcup_{n=1}^{\infty} A_n) = 0$