**Week 12 Assignment**

The due date for submitting this assignment has passed.

As per our records you have not submitted this assignment.

1) If \( W \) is the controllability Grammian matrix for the system

\[
x_1(k+1) = kx_2(k) - 2x_1(k) + w(k)
\]
\[
x_2(k+1) = x_1(k) + x_2(k) + 2a(k), \quad k = 0, 1, 2,
\]

then

\[
W = \begin{bmatrix}
\frac{10}{3} & -2 \\
\frac{10}{3} & 2
\end{bmatrix}
\]

Score: 0

No, the answer is incorrect.

Accepted Answers:

\[
W = \begin{bmatrix}
\frac{10}{3} & -2 \\
\frac{10}{3} & 2
\end{bmatrix}
\]

2) If \( M \) is the observability Grammian matrix for the system

\[
x_1(k+1) = x_1(k) + kx_3(k)
\]
\[
x_2(k+1) = x_2(k) + u(k),
\]

with the observation

\[
y(k) = x_1(k) - 2x_2(k), \quad k = 0, 1, 2,
\]

then

\[
M = \begin{bmatrix}
2 & -10 \\
-10 & 72
\end{bmatrix}
\]

Score: 0

No, the answer is incorrect.

Accepted Answers:

\[
M = \begin{bmatrix}
2 & -10 \\
-10 & 72
\end{bmatrix}
\]

3) Consider the systems

\[
x_1(k+1) = x_1(k) - 2x_2(k)
\]
\[
x_2(k+1) = -x_1(k) + x_2(k) + u(k)
\]

\[
y(k) = 2x_1(k) + x_2(k)
\]

and

\[
x_1(k+1) = 2x_2(k) - x_1(k)
\]
\[
x_2(k+1) = -x_1(k) + 2x_2(k) + u(k)
\]

\[
y(k) = 2x_1(k) + 3x_2(k)
\]

Then

- only first system is observable
- only second system is observable
- both the systems are observable
- none of them is observable
Consider the system
\[ x_1(k+1) = 2x_1(k) - x_2(k) \]
\[ x_2(k+1) = -x_1(k) + x_2(k) + u(k) \]
and
\[ x_1(k+1) = x_1(k) + \frac{1}{2}x_2(k) + u(k) \]
\[ x_2(k+1) = 2x_1(k) + x_2(k) + 2u(k) \]
then

- only first system is controllable
- only second system is controllable
- both the systems are controllable
- none of them is controllable

No, the answer is incorrect.
Score: 0

Consider the system
\[ x(k+1) = Ax(k), \quad x(k) \in \mathbb{R}^n \]
\[ x(k) = x_0 \]
where \( A_{n \times n} \) has \( n \) distinct eigen values \( \lambda_1, \lambda_2, \ldots, \lambda_n \) with the corresponding eigen vectors \( x_1, x_2, \ldots, x_n \), respectively. Then for nonzero scales:
\[ x(k) = \frac{1}{\lambda_1}x_1 + \frac{1}{\lambda_2}x_2 + \cdots + \frac{1}{\lambda_n}x_n \]
if
\[ x_1 = \frac{1}{\lambda_1}x_1 + \frac{1}{\lambda_2}x_2 + \cdots + \frac{1}{\lambda_n}x_n \]
\[ x_2 = \lambda_2x_1 + \frac{1}{\lambda_2}x_2 + \cdots + \frac{1}{\lambda_n}x_n \]
\[ x_3 = \lambda_3x_1 + \lambda_2x_2 + \cdots + \lambda_n x_n \]
\[ x_n = \lambda_n x_1 + \lambda_{n-1}x_2 + \cdots + \lambda_2 x_{n-1} + \lambda_1 x_n \]
No, the answer is incorrect.
Score: 0

The autonomous system
\[ x(k+1) = f(x(k)), \quad f(0) = 0 \]
is asymptotically stable at the trivial solution if there exists a Lyapunov function \( V(x(k)) \) such that
\[ V(x(k)) - V(x(k+1)) > 0 \]
\[ V(x(k+1)) - V(x(k)) \leq 0 \]
\[ V(x(k+1)) - V(x(k+1)) < 0 \]
\[ V(x(k+1)) - x(k) < 0 \]
No, the answer is incorrect.
Score: 0

The system
\[ x_1(k+1) = x_1(k) + 3x_2(k)x_2^2(k) \]
\[ x_2(k+1) = 3x_1(k)x_2(k) - x_2(k) \]
is unstable at \((0,0)\)
- is stable but not asymptotically stable at \((0,0)\)
- is asymptotically stable at \((0,0)\) with the Lyapunov function \( V(x_1(k), x_2(k)) = x_1^2(k) + x_2^2(k) \), if \( x_1^2(k) + x_2^2(k) < \frac{1}{2} \)
- is asymptotically stable at \((0,0)\) with the Lyapunov function \( V(x_1(k), x_2(k)) = x_1^2(k) + x_2^2(k) \), if \( x_1^2(k) + x_2^2(k) < 1 \)
No, the answer is incorrect.
Score: 0

If \( A = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix} \) and \( B = \begin{bmatrix} 1 \\ 1 \end{bmatrix} \), then
- the rows of \( e^{At} \) are linearly independent
- the rows of \( e^{Bt} \) are linearly independent functions of \( t \)
- the rows of \( e^{Bt} \) are linearly dependent functions of \( t \)
- the rows of \( (I - A)^{-1} \) are linearly independent functions of \( t \)
No, the answer is incorrect.
Score: 0
9) The control system
\[ \begin{align*}
    x_1 &= 2x_1 - 2x_2 + u \\
    x_2 &= -x_1 + 2x_2 + w
\end{align*} \]

is observable but the corresponding canonical system is not observable
is not observable but the corresponding canonical system is observable
and the corresponding canonical system are observable
is not controllable

No, the answer is incorrect.
Score: 0
Accepted answers:
and the corresponding canonical system are observable

10) Consider the continuous system
\[ \begin{align*}
    x(t) &= x(t) \\
    \dot{x}_1 &= -4x_1 + u
\end{align*} \]
and the corresponding discrete system
\[ \begin{align*}
    x(k + 1) &= Ex(k) + Fu(k), \quad k = n_0, n_0 + 1, \ldots
\end{align*} \]
where \( x(k) = x(t_k), \ u(k) = u(t_k), \ t_k = kh, \ E = e^{kh} \) and \( F = \left( \int_0^1 e^{kh} \, dh \right) B, \ A = \begin{bmatrix} 0 & 1 \\ -4 & 0 \end{bmatrix}, \ B = \begin{bmatrix} 0 \\ 1 \end{bmatrix}, \ h > 0. \) Then

the continuous system is not controllable
the continuous system is controllable but the corresponding discrete system is not controllable for \( h = \pi/4 \)
the corresponding discrete system is controllable for \( h = \pi/2 \)
the corresponding discrete system is controllable for \( h = \pi/8 \)

No, the answer is incorrect.
Score: 0
Accepted answers:
the corresponding discrete system is controllable for \( h = \pi/8 \)