### Assignment 4

The due date for submitting this assignment has passed.
As per our records you have not submitted this assignment.

1) Consider the differential equations

\[ t^3y'' + (\sin t)y' + ty = 0 \quad (1) \]
\[ t^3y'' + (\sin t)y' + ty = 0 \quad (2) \]

Then the point \( t = 0 \) is

- An ordinary point for both the equations
- A regular singular point of (1) but not of (2)
- A irregular singular point for both the equations
- A regular singular point of (2) but not of (1)

No, the answer is incorrect.
Score: 0

Accepted Answers:
A regular singular point of (1) but not of (2)

2) Consider the differential equation \( t(t - 1)^2 (t + 3)x'' + t^2x' - (t^2 + t - 1)x = 0 \). Then

Points \( t = 0, t = 1 \) and \( t = -3 \) all are regular singular points
- Only point \( t = 0 \) is a regular singular point
- Only point \( t = 1 \) is a regular singular point
- Points \( t = 0 \) and \( t = -3 \) are regular singular points

No, the answer is incorrect.
Score: 0

Accepted Answers:
Points \( t = 0 \) and \( t = -3 \) are regular singular points

3) The solution of the differential equation \( t^2y'' + 3ty' + y = 0 \) with \( y(1) = 0 \) and \( y(e) = 1 \) is

1 point
$$y(t) = e^t \ln t$$

None of these

No, the answer is incorrect.
Score: 0

Accepted Answers:

$$y(t) = \frac{e^t}{t} \ln t$$

4) Consider the differential equation $2ty'' + y' + ty = 0$, $0 < t < \infty$.
Then the recurrence relation to obtain Frobenius series solution
$$y(t) = t^r \sum_{n=0}^{\infty} a_n t^n$$

For $r = 0$, $n(2n+1)a_n + a_{n-2} = 0$, $n \geq 2$

For $r = \frac{1}{2}$, $n(2n-1)a_n + a_{n-2} = 0$, $n \geq 2$

For $r = 1$, $n(2n-1)a_n + a_{n-2} = 0$, $n \geq 2$

For $r = \frac{3}{2}$, $n(2n+1)a_n + a_{n-2} = 0$, $n \geq 2$

No, the answer is incorrect.
Score: 0

Accepted Answers:

For $r = \frac{3}{2}$, $n(2n+1)a_n + a_{n-2} = 0$, $n \geq 2$

5) Consider the differential equation $2t^2y'' + t(2t+1)y' - y = 0$.
Then differential equation has

only one Frobenius series solution

two Frobenius series solutions and one of the Frobenius series solution is given by $y(t) = t^{-1/2}[1 - \frac{3}{5}t^2 + \frac{4}{15}t^3 - \ldots]$

two Frobenius series solutions and one of the Frobenius series solution is given by $y(t) = t[1 - t + \frac{1}{2}t^2 - \ldots]$

two Frobenius series solutions and one of the Frobenius series solution is given by $y(t) = [t - \frac{2}{5}t^2 + \frac{4}{15}t^3 - \ldots]$

No, the answer is incorrect.
Score: 0

Accepted Answers:

two Frobenius series solutions and one of the Frobenius series solution is given by $y(t) = [t - \frac{2}{5}t^2 + \frac{4}{15}t^3 - \ldots]$

6) Consider the differential equation $t^2y'' + (3t - t^2)y' - ty = 0$. Then

roots of the indicial equation differ by a positive integer and only one Frobenius series solution exist

roots of the indicial equation differ by a positive integer and both Frobenius series solutions exist

roots of the indicial equation differ by a positive integer and only one Frobenius series solution exist and is given by
$$y(t) = t[1 - t + \frac{1}{2}t^2 - \ldots]$$
roots of the indicial equation differ by a positive integer and both 
Frobenius series solutions exist and one of the solution is given by 
y(t) = t[1 - t + \frac{1}{3} t^2 - ...]

No, the answer is incorrect.
Score: 0
Accepted Answers:
roots of the indicial equation differ by a positive integer and both Frobenius series solutions exist

7) Consider the differential equation \( t^2 y'' + (t^2 - 3t) y' + 3y = 0 \). Then 

- roots of the indicial equation are 1 and 2
- roots of the indicial equation are 0 and 3
- roots of the indicial equation differ by a positive integer and has only 
one Frobenius series solution exist and is given by \( y(t) = t^3 e^{-t} \)
- roots of the indicial equation differ by a positive integer and has two 
Frobenius series solutions and one of the solution is given by \( y(t) = t e^{-t} \)

No, the answer is incorrect.
Score: 0
Accepted Answers:
roots of the indicial equation differ by a positive integer and has only 
one Frobenius series solution exist and is given by \( y(t) = t^3 e^{-t} \)

8) Consider the differential equation \( (t - t^2)y'' + (1 - 5t)y' - 4y = 0 \). Then 

- Roots of the indicial equation are unequal
- Roots of the indicial equation are equal and one of the solution is 
given as \( y(t) = \sum_{n=0}^{\infty} n^2 t^n \)
- Recurrence relation to find the coefficients \( \{a_n\} \) is \( a_n = \frac{(k+n)^2}{(k+1)^2} a_{n-1} \)
- The general solution of the differential equation is given by 
y(t) = c_1 y_1 + c_2 (y_1 \ln t - 2(2x + 6x^2 + 12x^3 + ...)), 
where \( y_1 \) is Frobenius series solution of the given equation

No, the answer is incorrect.
Score: 0
Accepted Answers:
The general solution of the differential equation is given by 
y(t) = c_1 y_1 + c_2 (y_1 \ln t - 2(2x + 6x^2 + 12x^3 + ...)), 
where \( y_1 \) is Frobenius series solution of the given equation

9) Consider the differential equation \( (x - x^2)y'' + (1 - x)y' - y = 0 \) near 0 and let its Frobenius 
solution is given as \( y(x) = \sum_{n=0}^{\infty} a_n x^{n+r} \). Then 

- Roots of the indicial equation are unequal and one of the solution is given by 
y(x) = a \left( 1 + x + \frac{1}{3} x^2 + \frac{19}{30} x^3 + ... \right), \text{ where } a \text{ is constant.}
Roots of the indicial equation differs by a positive integer.

it has only one Frobenius series solution and is given by

\[ y(x) = a \left(1 + x + \frac{1}{2} x^2 + \frac{1}{15} x^3 + \ldots\right) \]

The general solution of the differential equation is given by

\[ y(x) = a \ln x \left(1 + x + \frac{1}{2} x^2 + \frac{10}{21} x^3 + \ldots\right) + b(-2x - x^2 - \ldots), \text{ where } a \text{ and } b \text{ are constants} \]

No, the answer is incorrect.

Score: 0

Accepted Answers:

The general solution of the differential equation is given by

\[ y(x) = a \ln x \left(1 + x + \frac{1}{2} x^2 + \frac{10}{21} x^3 + \ldots\right) + b(-2x - x^2 - \ldots), \text{ where } a \text{ and } b \text{ are constants.} \]

10) The solution of the initial value problem \( t^2 y'' - ty' - 2y = 0 \), \( y(1) = 1 \) on the interval \( 0 < t < \infty \) is

\[ \frac{t}{2\sqrt{3}} \left( t\sqrt{3} + \frac{1}{\sqrt{3}} - 2 \right) \]

No, the answer is incorrect.

Score: 0

Accepted Answers:

\[ \frac{t}{2\sqrt{3}} \left( t\sqrt{3} - \frac{1}{\sqrt{3}} \right) \]