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NPTEL reviewer3@nptel.iitm.ac.in ▾

Courses » Ordinary and Partial Differential Equations and Applications

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Unit 13 - Week 12

Assignment 12

The due date for submitting this assignment has passed. **Due on 2018-10-24, 23:59 IST.**
As per our records you have not submitted this assignment.

1) *1 point*

The solution $u(x,t)$ *of the heat equation* $u_t = \kappa(u_{xx} + u_{yy}), 0 \le x \le \pi, 0 \le y \le \pi$
$u(x,y,0) = xy, 0 \le x \le \pi, 0 \le y \le \pi$
$u(0,y,t) = u(\pi,y,t) = u(x,0,t) = u(x,\pi,t) = 0$
is given by

- $4\sum_{m=1}^{\infty}\sum_{n=1}^{\infty}\frac{(-1)^{m+n}}{mn}e^{-\kappa t(m^2+n^2)}\sin mx \sin ny$
- $\sum_{m=1}^{\infty}\sum_{n=1}^{\infty}\frac{(-1)^{m+n}}{mn}e^{-\kappa t(m+n)}\sin mx \sin ny$
- $4\sum_{m=1}^{\infty}\sum_{n=1}^{\infty}\frac{(-1)^{m+n}}{mn}e^{-\kappa t(m^2+n^2)}\sin mx \cos ny$
- $4\sum_{m=1}^{\infty}\sum_{n=1}^{\infty}\frac{(-1)^{m+n}}{mn}e^{-\kappa t(m+n)}\sin ny \cos mx$

No, the answer is incorrect.
Score: 0

Accepted Answers:
$4\sum_{m=1}^{\infty}\sum_{n=1}^{\infty}\frac{(-1)^{m+n}}{mn}e^{-\kappa t(m^2+n^2)}\sin mx \sin ny$

2) *1 point*

Consider the following two dimensional wave equation defined on a rectanglular domain as follows :
$u_{tt} - c^2(u_{xx} + u_{yy}) = 0, 0 \le x \le \pi, 0 \le y \le \pi$
with boundary conditions $u(0,y,t) = u(\pi,y,t) = u(x,0,t) = u(x,\pi,t) = 0$ *and initial condi*
$u(x,y,0) = \sin x \sin y$ *and* $u_t(x,y,0) = 0.$
Then the solution $u(x,t)$ *is given by*

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$$\sum_{m=1}^{\infty}\sum_{n=1}^{\infty}\frac{1}{c\sqrt{m^2+n^2}}\sin(m^2+n^2)ct\sin mx\sin ny$$

$None\ of\ these.$

No, the answer is incorrect.
Score: 0

Accepted Answers:
$\cos\sqrt{2}ct\sin x\sin y$

3) **1 point**

$A\ solution\ of\ the\ following\ heat\ equation\ defined\ in\ the\ region\ 0\ \leq\ x\ \leq\ \pi,\ t\ \geq\ 0$

$u_t=\kappa u_{xx},\quad 0<x<\pi, t>0$

$u(0,t)=u(\pi,t)=0$

$$u(x,0)=\begin{cases}x, & 0\leq x\leq\pi/2;\\ \pi-x, & \pi/2\leq x\leq\pi.\end{cases}$$

$Then\ u(x,t)\ is$

$$\sum_{n=1}^{\infty}\frac{e^{-\kappa n^2t}\sin(n\pi/2)}{n^2}\sin nx$$

$$\frac{4}{\pi}\sum_{n=1}^{\infty}\frac{e^{-\kappa n^2t}\cos(n\pi/2)}{n^2}\sin nx$$

$$\frac{4}{\pi}\sum_{n=1}^{\infty}\frac{e^{-\kappa n^2t}\sin(n\pi/2)}{n^2}\sin nx$$

$$\sum_{n=1}^{\infty}\frac{e^{-\kappa n^2t}\cos(n\pi/2)}{n^2}\sin nx$$

No, the answer is incorrect.
Score: 0

Accepted Answers:

$$\frac{4}{\pi}\sum_{n=1}^{\infty}\frac{e^{-\kappa n^2t}\sin(n\pi/2)}{n^2}\sin nx$$

4) $Let\ u(x,t)\ be\ a\ solution\ of\ the\ following\ heat\ equation\ in\ an\ infinite\ rod:$ **1 point**

$u_t=\kappa u_{xx},\quad -\infty<x<\infty$

$$u(x,0)=\begin{cases}A_0, & a<x<b\,;\\ 0, & x\leq a \text{ or } y\geq b.\end{cases}$$

$Then\ u(x,t)\ is\ given\ by$

$$A_0\left[erf\left(\frac{b+x}{2\sqrt{\kappa t}}\right)-erf\left(\frac{a+x}{2\sqrt{\kappa t}}\right)\right]$$

$$\frac{A_0}{2}\left[erf\left(\frac{b+x}{2\sqrt{\kappa t}}\right)-erf\left(\frac{a+x}{2\sqrt{\kappa t}}\right)\right]$$

$$\frac{A_0}{2}\left[erf\left(\frac{b-x}{2\sqrt{\kappa t}}\right)-erf\left(\frac{a-x}{2\sqrt{\kappa t}}\right)\right]$$

$$A_0\left[erf\left(\frac{b-x}{2\sqrt{\kappa t}}\right)-erf\left(\frac{a-x}{2\sqrt{\kappa t}}\right)\right]$$

No, the answer is incorrect.

Score: 0

Accepted Answers:

$\frac{A_0}{2}\left[erf\left(\frac{b-x}{2\sqrt{\kappa t}}\right) - erf\left(\frac{a-x}{2\sqrt{\kappa t}}\right)\right]$

5) $Let\ u(x,t)\ be\ a\ solution\ of\ the\ following\ heat\ equation\ in\ a\ semi\ infinite\ rod\ which\ is\ given\ as$ **1 point**

$u_t = \kappa u_{xx},\quad x \geq 0, t \geq 0$

$u(0,t) = u_0,\quad t \geq 0$

$u(x,0) = 0,\quad 0 < x < \infty,$

$Also,\ u\ and\ u_x\ \to 0\ as\ x \to \infty\ and\ it\ is\ given\ that$

$\frac{2}{\sqrt{\pi}}\int_y^\infty e^{-x^2}dx = erfc(y) = 1 - erf(y),$

$\int_0^\infty \frac{\sin ax}{x}\,dx = \pi/2,$

$\int_0^\infty e^{-\omega^2}\frac{\sin 2\omega z}{\omega}\,d\omega = \frac{\pi}{2}\,erf(z).$

$Then\ u(x,t)\ is\ equal\ to$

- $u_0\left[erfc\left(\frac{x}{2\sqrt{\kappa t}}\right) - 1\right]$
- $u_0\left[erfc\left(\frac{x}{2\sqrt{\kappa t}}\right)\right]$
- $u_0\left[erfc\left(\frac{x}{\sqrt{\kappa t}}\right)\right]$
- $u_0\left[erfc\left(\frac{x}{\sqrt{\kappa t}}\right) - 1\right]$

No, the answer is incorrect.
Score: 0

Accepted Answers:

$u_0\left[erfc\left(\frac{x}{2\sqrt{\kappa t}}\right)\right]$

6) $The\ solution\ u(x,t)\ of\ the\ wave\ equation\ u_{tt} = u_{xx} + t\sin x, 0 \leq x \leq \pi, t > 0$ **1 point**

$u(0,t) = u(\pi,t) = 0$

$u(x,0) = u_t(x,0) = 0$

is

- $\sum_{n=1}^{\infty}\frac{1}{n^3}\sin nt \sin nx$
- $\sum_{n=1}^{\infty}\frac{1}{n^2}(n - \sin nt)\sin nx$
- $(t - \sin t)\sin x$
- $none\ of\ these.$

No, the answer is incorrect.
Score: 0

Accepted Answers:

$(t - \sin t)\sin x$

7) **1 point**

Consider the wave equation $u_{tt} = c^2 u_{xx}, 0 < x < 1$ *with boundary conditions* $u(0,t) = t^2,\ u(1,t) = \sin t$ *and initial conditions* $u(x,0) = u_t(x,0) = 0.$
An equivalent formulation is given as $v_{tt} = c^2 v_{xx} + F(x,t)$
with boundary conditions $v(0,t) = 0,\ v(1,t) = 0$ *and initial conditions* $v(x,0) = f(x),\ v_t(x,0) = g(x),$ *where*

- $F(x,t) = 2x + x\sin t$
- $f(t) = -t + \sin t$
- $g(x) = x$
- $F(x,t) = 2(x-1) + x\sin t$

No, the answer is incorrect.
Score: 0

Accepted Answers:
$F(x,t) = 2(x-1) + x\sin t$

8) ***1 point***

The solution of the initial value problem $u_{tt} = u_{xx} + \pi^2 \sin \pi x, 0 < x < 1, t > 0$ *with initial conditions* $u(x,0) = u_t(x,0) = 0$ *and boundary conditions* $u(0,t) = u(1,t) = 0$ *is given by*

- $\sin \pi x - \cos \pi t$
- $(1 - \sin \pi x)\cos \pi t$
- $(1 - \cos \pi t)\sin \pi x$
- $(1 - \sin \pi t)\sin \pi x$

No, the answer is incorrect.
Score: 0

Accepted Answers:
$(1 - \cos \pi t)\sin \pi x$

9) *The solution of the heat equation* $u_t = u_{xx} + x(\pi - x), 0 < x < \pi, t > 0$ ***1 point***

$u(0,t) = u(\pi,t) = 0$
$u(x,0) = 0$

using Duhamel's principle is given by $u(x,t) = \sum_{n=1}^{\infty} \int_0^t a_n(\tau) e^{-n^2(t-\tau)} d\tau \sin nx$

where $a_n(\tau)$ *is*

- $\begin{cases} \frac{4}{n^3}, & \text{n is odd;} \\ 0, & \text{n is even.} \end{cases}$
- $\begin{cases} \frac{8}{\pi n^3}, & \text{n is odd;} \\ 0, & \text{n is even.} \end{cases}$
- $\begin{cases} \frac{4}{n^3}, & \text{n is even;} \\ 0, & \text{n is odd.} \end{cases}$
-

$\begin{cases} \frac{8}{\pi n^3}, & \text{n is even;} \\ 0, & \text{n is odd.} \end{cases}$

No, the answer is incorrect.
Score: 0

Accepted Answers:

$\begin{cases} \frac{8}{\pi n^3}, & \text{n is odd;} \\ 0, & \text{n is even.} \end{cases}$

10) *1 point*

The solution of the heat equation

$u_t = u_{xx} - x^2, 0 < x < \pi, t > 0$

$u(0,t) = 0, t \geq 0$

$u(\pi,t) = -\pi^2 t, t \geq 0$

$u(x,0) = 0$

$\text{using Duhamel's principle is given by } u(x,t) = \sum_{n=1}^{\infty} \int_0^t a_n(\tau) e^{-n^2(t-\tau)} d\tau \sin nx \text{ where } a_n(\tau)$

- $\begin{cases} \frac{4}{n^3}, & \text{n is odd;} \\ 0, & \text{n is even.} \end{cases}$
- $\begin{cases} \frac{8}{\pi n^3}, & \text{n is odd;} \\ 0, & \text{n is even.} \end{cases}$
- $\begin{cases} \frac{4}{n^3}, & \text{n is even;} \\ 0, & \text{n is odd.} \end{cases}$
- $\begin{cases} \frac{8}{\pi n^3}, & \text{n is even;} \\ 0, & \text{n is odd.} \end{cases}$

No, the answer is incorrect.
Score: 0

Accepted Answers:

$\begin{cases} \frac{8}{\pi n^3}, & \text{n is odd;} \\ 0, & \text{n is even.} \end{cases}$

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