Courses » Ordinary and Partial Differential Equations and Applications

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Unit 11 - Week 10

Assignment 10

The due date for submitting this assignment has passed.
As per our records you have not submitted this assignment.

Due on 2018-10-10, 23:59 IST.

1) *1 point*

The steady state temperature in a circular plate of radius a which has one half $(0 < \theta < \pi)$ of its circumference at a constant temperature u_0 and the other half at temperature $0^\circ C$ is given by

- $u(r,\theta) = \frac{2u_0}{\pi} \sum_{n=1}^{\infty} \frac{1}{n} \left(\frac{r}{a}\right)^n \sin n\theta$
- $u(r,\theta) = \frac{2u_0}{\pi} \sum_{n=1}^{\infty} \frac{1}{(2n-1)} \left(\frac{r}{a}\right)^{2n-1} \sin(2n-1)\theta$
- $u(r,\theta) = \frac{u_0}{2} + \frac{2u_0}{\pi} \sum_{n=1}^{\infty} \frac{1}{n} \left(\frac{r}{a}\right)^n \sin n\theta$
- $u(r,\theta) = \frac{u_0}{2} + \frac{2u_0}{\pi} \sum_{n=1}^{\infty} \frac{1}{(2n-1)} \left(\frac{r}{a}\right)^{2n-1} \sin(2n-1)\theta$

No, the answer is incorrect.
Score: 0

Accepted Answers:
$u(r,\theta) = \frac{u_0}{2} + \frac{2u_0}{\pi} \sum_{n=1}^{\infty} \frac{1}{(2n-1)} \left(\frac{r}{a}\right)^{2n-1} \sin(2n-1)\theta$

2) *1 point*

A thin uniformly long metal plate is in the form of area enclosed by the lines $y = 0$, $y = b$, $x = 0$ and extending to infinity on the positive side of $x-$axis. The temperature is kept at $0\,^o\,C$ along the edges $y = 0$, $y = b$ and at infinity throughout. if the e

- $u(r,\theta) = \frac{4u_0}{\pi} \sum_{n=1}^{\infty} \frac{1}{n} e^{-n\pi x/b} \sin \frac{n\pi y}{b}$
- $u(r,\theta) = \frac{4u_0}{\pi} \sum_{n=1}^{\infty} \frac{1}{(2n-1)} e^{-(2n-1)\pi x/b} \sin \frac{(2n-1)\pi y}{b}$
- $u(r,\theta) = \frac{2u_0}{\pi} \sum_{n=1}^{\infty} \frac{1}{n} e^{-n\pi x/b} \sin \frac{2n\pi y}{b}$
- $u(r,\theta) = \frac{4u_0}{\pi} \sum_{n=1}^{\infty} \frac{1}{n} e^{-n\pi x/b} \cos \frac{n\pi y}{b}$

No, the answer is incorrect.
Score: 0

Accepted Answers:
$u(r,\theta) = \frac{4u_0}{\pi} \sum_{n=1}^{\infty} \frac{1}{(2n-1)} e^{-(2n-1)\pi x/b} \sin \frac{(2n-1)\pi y}{b}$

3) *1 point*

If $\lambda_n = \frac{\pi\, n}{a}$ and $\mu_m = \frac{\pi\, m}{b}$ then the potential function $\psi(x,y,z)$ in the region $0 \le x \le a$, $0 \le y \le b$, $0 \le z \le c$ satisfying the conditions $\psi = 0$, on $x = 0$, $x = a$, $y = 0$, $y = b$, $z = 0$ and $\psi = f(x,y)$ on $z = c$, $0 \le x \le a$, $0 \le y \le b$ is given by

- $\psi(x,y,z) = \sum_{n=1}^{\infty} \sum_{m=1}^{\infty} E_{nm} \sin \frac{n\pi x}{a} \sin \frac{m\pi y}{b} \sinh(\sqrt{\lambda_n^2 + \mu_m^2})z$
  where $E_{nm} = \frac{4}{ab} \frac{\int_0^a \int_0^b f(x,y) \sin \frac{n\pi x}{a} \sin \frac{m\pi y}{b}\, dydx}{\sinh(\sqrt{\lambda_n^2 + \mu_m^2})c}$
- $\psi(x,y,z) = \sum_{n=1}^{\infty} \sum_{m=1}^{\infty} E_{nm} \sin \frac{n\pi x}{a} \sin \frac{m\pi y}{b} \sinh(\sqrt{\lambda_n^2 + \mu_m^2})z$
  where $E_{nm} = \frac{4}{ab} \int_0^a \int_0^b f(x,y) \sin \frac{n\pi x}{a} \sin \frac{m\pi y}{b}\, dydx$
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  where $E_{nm} = \frac{\int_0^a \int_0^b f(x,y) \sin \frac{n\pi x}{a} \sin \frac{m\pi y}{b}\, dydx}{\sinh(\sqrt{\lambda_n^2 + \mu_m^2})c}$
- $\psi(x,y,z) = \sum_{n=1}^{\infty} \sum_{m=1}^{\infty} E_{nm} \sin \frac{n\pi x}{a} \sin \frac{m\pi y}{b} \cosh(\sqrt{\lambda_n^2 + \mu_m^2})z$
  where $E_{nm} = \frac{4}{ab} \frac{\int_0^a \int_0^b f(x,y) \sin \frac{n\pi x}{a} \sin \frac{m\pi y}{b}\, dydx}{\cosh(\sqrt{\lambda_n^2 + \mu_m^2})c}$

No, the answer is incorrect.
Score: 0

Accepted Answers:
$\psi(x,y,z) = \sum_{n=1}^{\infty} \sum_{m=1}^{\infty} E_{nm} \sin \frac{n\pi x}{a} \sin \frac{m\pi y}{b} \sinh(\sqrt{\lambda_n^2 + \mu_m^2})z$
where $E_{nm} = \frac{4}{ab} \frac{\int_0^a \int_0^b f(x,y) \sin \frac{n\pi x}{a} \sin \frac{m\pi y}{b}\, dydx}{\sinh(\sqrt{\lambda_n^2 + \mu_m^2})c}$

4) *The potential in the interior of a sphere of unit radius when the potential on the surface is $f(\theta) = \cos 2\theta$, is* *1 point*

- $u(r,\theta) = r^2\left(\cos^2\theta - \frac{1}{3}\right) - \frac{2}{3}$
- $u(r,\theta) = 2r^2\left(\cos^2\theta - \frac{1}{3}\right) - \frac{2}{3}$
- $u(r,\theta) = 2r^2\left(\cos^2\theta - \frac{1}{3}\right) - \frac{1}{3}$
- $u(r,\theta) = r^2\left(\cos^2\theta - \frac{1}{3}\right) - \frac{1}{3}$

No, the answer is incorrect.
Score: 0

Accepted Answers:
$u(r,\theta) = 2r^2\left(\cos^2\theta - \frac{1}{3}\right) - \frac{1}{3}$

5) *1 point*

The steady state temperature distribution between two coaxial circular cylinders of radii r_0 and r_1, kept at the temperatures u_0 and u_1, respectively, is given by

- $u = \left\{(u_1 - u_0)\ln r + u_0 \ln r_1 - u_1 \ln r_0\right\} / \ln\left(\frac{r_1}{r_0}\right)$
- $u = \left\{(u_0 - u_1)\ln r + u_0 \ln r_1 - u_1 \ln r_0\right\} / \ln\left(\frac{r_1}{r_0}\right)$
- $u = \left\{(u_1 - u_0)\ln r + u_1 \ln r_0 - u_0 \ln r_1\right\} / \ln\left(\frac{r_1}{r_0}\right)$
- $u = \left\{(u_0 - u_1)\ln r + u_1 \ln r_0 - u_0 \ln r_1\right\} / \ln\left(\frac{r_1}{r_0}\right)$

No, the answer is incorrect.
Score: 0

Accepted Answers:
$u = \left\{(u_1 - u_0)\ln r + u_0 \ln r_1 - u_1 \ln r_0\right\} / \ln\left(\frac{r_1}{r_0}\right)$

6) *1 point*

The potential outside a spherical surface which is kept at a fixed distribution of electrical potential $u = f(\theta)$, is given by

- $u(r,\theta) = \sum_{n=0}^{\infty} \frac{(2n+1)}{2} \left(\frac{a}{r}\right)^{n+1} (\int_0^\pi f(\theta) P_n(\cos\theta) \sin\theta\, d\theta) P_n(\cos\theta)$
- $u(r,\theta) = \sum_{n=0}^{\infty} \frac{(2n+1)}{2} \left(\frac{a}{r}\right)^{n+1} (\int_0^\pi f(\theta) P_n(\cos\theta)\, d\theta) P_n(\cos\theta)$
- $u(r,\theta) = \sum_{n=0}^{\infty} \left(\frac{a}{r}\right)^{n+1} (\int_0^\pi f(\theta) P_n(\cos\theta) \sin\theta\, d\theta) P_n(\cos\theta)$
- $u(r,\theta) = \sum_{n=0}^{\infty} \left(\frac{a}{r}\right)^{n+1} (\int_0^\pi f(\theta) P_n(\cos\theta)\, d\theta) P_n(\cos\theta)$

No, the answer is incorrect.
Score: 0

Accepted Answers:
$u(r,\theta) = \sum_{n=0}^{\infty} \frac{(2n+1)}{2} \left(\frac{a}{r}\right)^{n+1} (\int_0^\pi f(\theta) P_n(\cos\theta) \sin\theta\, d\theta) P_n(\cos\theta)$

7) *1 point*

The steady state temperature distribution inside a long circular cylinder of two halves maintained at temperatures u_1 and u_2 respectively, is given by

- $u(r,\theta) = \frac{u_1 + u_2}{2} + \frac{2(u_1 - u_2)}{\pi} \sum_{n=1}^{\infty} \left(\frac{r}{a}\right)^{2n-1} \frac{\sin\,(2n-1)\theta}{(2n-1)}$
- $u(r,\theta) = \frac{u_1 + u_2}{2} + \frac{(u_1 - u_2)}{\pi} \sum_{n=1}^{\infty} \left(\frac{r}{a}\right)^{2n} \frac{\sin 2n\theta}{2n}$
- $u(r,\theta) = \frac{u_1 + u_2}{2} + \frac{2(u_1 - u_2)}{\pi} \sum_{n=1}^{\infty} \left(\frac{r}{a}\right)^{n} \frac{\sin n\theta}{n}$
- $u(r,\theta) = \frac{u_1 + u_2}{2} + \frac{(u_1 - u_2)}{\pi} \sum_{n=1}^{\infty} \left(\frac{r}{a}\right)^{n} \frac{\sin n\theta}{n}$

No, the answer is incorrect.
Score: 0

Accepted Answers:
$u(r,\theta) = \frac{u_1 + u_2}{2} + \frac{2(u_1 - u_2)}{\pi} \sum_{n=1}^{\infty} \left(\frac{r}{a}\right)^{2n-1} \frac{\sin\,(2n-1)\theta}{(2n-1)}$

8) *1 point*

A rectangular plate with insulated surface is 10 cm. wide and so long compared to its width that it may be considered infinite in length. If the temperature of the short edge $y = 0$ is given by

$$u = \begin{cases} 20x, 0 \le x \le 5 \\ 20(10 - x),\ 5 \le x \le 10 \end{cases}$$

and the two long edges $x = 0$, $x = 10$ as well as the other short edge are kept at $0^\circ C$ then the temperature $u(x,y)$ in the steady state is given by

- $u(x,y) = \frac{800}{\pi^2} \sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{(2n-1)^2} \sin \frac{(2n-1)\pi\, x}{10}\, e^{-\frac{(2n-1)\pi\, y}{10}}$
- $u(x,y) = \frac{800}{\pi^2} \sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{n^2} \sin \frac{n\pi\, x}{10}\, e^{-\frac{n\pi\, y}{10}}$
- $u(x,y) = \frac{200}{\pi^2} \sum_{n=1}^{\infty} \frac{1}{n^2} \sin \frac{n\pi\, x}{5}\, e^{-\frac{n\pi\, y}{5}}$
-

$u(x,y) = \frac{800}{\pi^2} \sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{(2n-1)^2} \sin \frac{(2n-1)\pi x}{10} e^{-\frac{(2n-1)\pi y}{10}}$

9) **1 point**

The bounding diameter of a semi – circular plate of radius 'a' cm. is kept at $0^{\circ}C$ and the temperature along the semi – circular boundary is given by

$$u(a,\theta) = \begin{cases} 50\theta, 0 < \theta \le \frac{\pi}{2} \\ 50(\pi - \theta), \frac{\pi}{2} < \theta < \pi \end{cases}.$$

Then the steady state temperature function $u(r,\theta)$ is given by

- $u(r,\theta) = \frac{50}{\pi} \sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{n^2} \left(\frac{r}{a}\right)^{n} \sin n\theta$
- $u(r,\theta) = \frac{200}{\pi} \sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{n^2} \left(\frac{r}{a}\right)^{n} \sin n\theta$
- $u(r,\theta) = \frac{50}{\pi} \sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{n^2} \left(\frac{r}{a}\right)^{2n} \sin 2n\theta$
- $u(r,\theta) = \frac{200}{\pi} \sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{(2n-1)^2} \left(\frac{r}{a}\right)^{2n-1} \sin (2n-1)\theta$

No, the answer is incorrect.
Score: 0

Accepted Answers:
$u(r,\theta) = \frac{200}{\pi} \sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{(2n-1)^2} \left(\frac{r}{a}\right)^{2n-1} \sin (2n-1)\theta$

10) *Let the Fourier transform of $f(x)$ be $F(s)$. Then the solution of equation* **1 point**

$u_t = c^2 \frac{\partial u}{\partial x^2} \quad (t > 0)$

subject to the initial condition $u(x,0) = f(x), -\infty < x < \infty$ is

- $u(x,t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} F(s) e^{-isx}\, ds$
- $u(x,t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} F(s) e^{c^2 s^2 t} e^{isx}\, ds$
- $u(x,t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} F(s) e^{c^2 s^2 t} e^{-isx}\, ds$
- $u(x,t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} F(s) e^{-c^2 s^2 t} e^{-isx}\, ds$

No, the answer is incorrect.
Score: 0

Accepted Answers:
$u(x,t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} F(s) e^{-c^2 s^2 t} e^{-isx}\, ds$

11) *The solution of the equation* **1 point**

$u_t = c^2 u_{xx} (x > 0, t > 0)$

such that $u(0,t) = 0$ and $u(x,0) = u_0$ is given by

- $u(x,t) = u_0\, erf\left(\frac{2x}{c\sqrt{t}}\right)$
- $u(x,t) = u_0\, erf\left(\frac{x}{c\sqrt{t}}\right)$
- $u(x,t) = u_0\, erf\left(\frac{x}{2c\sqrt{t}}\right)$
- $u(x,t) = u_0\, erf\left(\frac{x}{2ct}\right)$

No, the answer is incorrect.
Score: 0

Accepted Answers:
$u(x,t) = u_0\, erf\left(\frac{x}{2c\sqrt{t}}\right)$

12) *Using the inversion formula for the Laplace transform, the solution of wave equation* **1 point**

$y_{tt} = c^2 y_{xx} \quad (x > 0, t > 0)$

subject to the conditions

$y(x,0) = 0, \frac{\partial}{\partial t} y(x,0) = 0, y(0,t) = f(t)$ and $\lim_{x \to \infty} y(x,t) = 0,$

is given by

- $y(x,t) = f\left(t + \frac{x}{c}\right)$
- $y(x,t) = f\left(t - \frac{x}{c}\right)$
- $y(x,t) = f(t + cx)$
- $y(x,t) = f(t - cx)$

No, the answer is incorrect.
Score: 0

Accepted Answers:
$y(x,t) = f\left(t - \frac{x}{c}\right)$

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