### Week 3 Assessment

The due date for submitting this assignment has passed. **Due on 2018-02-14, 23:59 IST.**

**Submitted assignment**

1. **Let $S$ be a subset of a finite dimensional vector space then which of the following statement is false?**

   - $S^\perp = \{S\}^\perp$  
   - $\mathcal{L}(S) = S^{\perp\perp}$
   - $S^\perp = S^{\perp\perp}$
   - $\mathcal{L}(S) = S$

   **No, the answer is incorrect.**
   **Score:** 0
   **Accepted Answers:**
   $\mathcal{L}(S) = S$

2. **Let $U = \{(x, y, 0) : x, y \in \mathbb{R}\}$ be a subspace of $\mathbb{R}^3$. Then $U^\perp$ is equal to**

   - $\{(0, y, z) : z \in \mathbb{R}\}$
   - $\{(0, y, z) : y, z \in \mathbb{R}\}$
   - $\{(x, 0, z) : x, z \in \mathbb{R}\}$
   - $\{(x, y, z) : x + y + z = 0\}$

   **No, the answer is incorrect.**
   **Score:** 0
   **Accepted Answers:**
   $\{(0, 0, z) : z \in \mathbb{R}\}$

3. **Let $S$ be a subspace of $\mathbb{R}^3$ given by $S = \text{span}\{(1, 0, 0)\}$. Then $S^\perp$ is equal to**

   - $\text{span}\{(1, 0, 0), (0, 1, 0)\}$
   - $\text{span}\{(1, 0, 0), (0, 0, 1)\}$
   - $\text{span}\{(0, 1, 0), (0, 0, 1)\}$
   - $\text{span}\{(1, 0, 0), (0, 1, 0), (0, 0, 1)\}$

   **No, the answer is incorrect.**
   **Score:** 0
   **Accepted Answers:**
No, the answer is incorrect. 
Score: 0 
Accepted Answers: 

5) Let \( A : \mathbb{R}^3 \to \mathbb{R}^4 \), where \( A = \begin{pmatrix} 1 & 2 & 3 \\ -1 & -2 & -3 \\ 1 & 3 & 4 \\ 1 & 4 & 5 \end{pmatrix} \) then the dimension of the image of \( A \) is given by

6) Let \( A : \mathbb{R}^3 \to \mathbb{R}^4 \), where \( A = \begin{pmatrix} 1 & 2 & 3 \\ -1 & -2 & -3 \\ 1 & 3 & 4 \\ 1 & 4 & 5 \end{pmatrix} \) then the dimension of \( \ker(A) \) is

7) Let \( F : \mathbb{R}^4 \to \mathbb{R}^3 \) be the linear mapping defined by \( F(x, y, z, t) = (x - y + z + t, x + 2z - t, x + y + 3z - 3t) \) then the dimension of the image of \( F \) and the kernel of \( F \) are given by

8) Linearly independent eigenvectors corresponding to the eigen value \( \lambda = 0 \) of multiplicity of two of \( A = \begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix} \)
9) Let \( A = \begin{pmatrix} 1 & -1 \\ 0 & 1 \end{pmatrix} \) then which of the following statements is false?

- The eigenvalues of \( A \) are 1, 1
- A linearly independent vector is \( \begin{pmatrix} 1 \\ 0 \end{pmatrix} \)
- \( A \) is non-singular
- \( A \) is diagonalizable

No, the answer is incorrect.
Score: 0
Accepted Answers:
\{ (1, 0, -1)^T, (1, -1, 0)^T \}

10) Two matrices are similar if

- they have the same eigenvalues
- they have the same determinant
- they have the same trace
- they represent the same linear operator

No, the answer is incorrect.
Score: 0
Accepted Answers:
they represent the same linear operator

11) Let \( A = \begin{pmatrix} 1 & 1 \\ -1 & 1 \end{pmatrix} \). Then which of the following statements is false?

- \( A \) is non-singular
- \( A \) is diagonalizable over the field of real numbers \( \mathbb{R} \)
- The characteristic equation of \( A \) has no roots in \( \mathbb{R} \)
- The characteristic polynomial of \( A \) is \( \lambda^2 - 2\lambda + 2 \)

No, the answer is incorrect.
Score: 0
Accepted Answers:
A is diagonalizable over the field of real numbers \( \mathbb{R} \)

12) Let \( A = \begin{pmatrix} 7 & 3 \\ 3 & -1 \end{pmatrix} \). Then an orthogonal matrix \( P \) which diagonalizes \( A \) is given by

\( \begin{pmatrix} 3 & 1 \\ 1 & -3 \end{pmatrix} \)
No, the answer is incorrect.
Score: 0
Accepted Answers:
\[
\begin{pmatrix}
\frac{3}{10} & \frac{1}{\sqrt{10}} \\
\frac{1}{\sqrt{10}} & -\frac{3}{10}
\end{pmatrix}
\]