Assignment 3

The due date for submitting this assignment has passed. Due on 2018-09-05, 23:59 IST.
As per our records you have not submitted this assignment.

1) The symmetric Fredholm integral equation \( y(x) = f(x) + \lambda \int_{-1}^{1} x^3 t^3 y(t) dt \), has

- no solution for \( \lambda = \frac{7}{2} \) and \( f(x) = x \)
- a unique solution for \( \lambda = \frac{2}{7} \) and \( f(x) = x \)
- a unique solution for \( \lambda = \frac{7}{2} \) and \( f(x) = x^2 \)
- no solution for \( \lambda = \frac{2}{7} \) and \( f(x) = x^2 \).

No, the answer is incorrect.
Score: 0
Accepted Answers:
a unique solution for \( \lambda = \frac{2}{7} \) and \( f(x) = x \)

2) The symmetric Fredholm integral equation \( y(x) = e^x + \lambda \int_{-1}^{1} (xt + 2t + 2x + 4) y(t) dt \), has

- no solution for \( \lambda = \frac{1}{9} \)
- a unique solution for \( \lambda = \frac{1}{9} \)
- infinitely many solutions for \( \lambda = \frac{3}{26} \)
- a unique solution for \( \lambda = \frac{3}{26} \).

No, the answer is incorrect.
Score: 0
Accepted Answers:
a unique solution for \( \lambda = \frac{1}{9} \)
Infinitely many solutions for $\lambda = 1$ and $f(x) = \cos x$

Unique solution for $\lambda = \pi$ and $f(x) = \sin x$.

No, the answer is incorrect.
Score: 0
Accepted Answers: 
unique solution for $\lambda = \pi$ and $f(x) = \sin x$.

4) Consider the Fredholm integral equation $y(x) = f(x) + \lambda \int_0^2 (\sin x + \sin t)g(t)dt$.
Then it has

- no solution for $\lambda = \frac{1}{\pi}$, $f(x) = \sin x$
- unique solution for $\lambda = \frac{1}{\pi^2}$, $f(x) = \sin x$
- unique solution for $\lambda = 1$, $f(x) = \cos x$
- infinitely many solutions for $\lambda = \frac{1}{\pi^2}$, $f(x) = \sin x$.

No, the answer is incorrect.
Score: 0
Accepted Answers: 
unique solution for $\lambda = 1$, $f(x) = \cos x$.

5) Consider the following symmetric Fredholm integral equation

$$\sin 4x = \lambda \int_0^{\pi/2} K(x, t)g(t)dt,$$
where
$$K(x, t) = \begin{cases} 
\sin x \cos t, & 0 \leq x \leq t; \\
\sin t \cos x, & t \leq x \leq \pi/2.
\end{cases}$$

Then it has

- no solution for all value of $\lambda$
- unique solution $y(x) = \sin 4x$ for only one value of $\lambda$
- unique solution $y(x) = \sin 4x$ for only two value of $\lambda$
- None of these.

No, the answer is incorrect.
Score: 0
Accepted Answers: 
unique solution $y(x) = \sin 4x$ for only one value of $\lambda$.

6) Consider the following Fredholm integral equation $y(x) = f(x) + \lambda \int_0^{2\pi} \sin(x + t)y(t)dt$. Th
using Hilbert Schmidt theorem we obtain

$$y(x) = x + \frac{1}{4} (2 \cos x + \sin x)$$ for $\lambda = \frac{2}{\pi}$ and $f(x) = x$.

$$y(x) = x + 2 \cos x + \sin x$$ for $\lambda = \frac{1}{\pi}$ and $f(x) = x$.

$$y(x) = 1 + \sin x + \cos x$$ for $\lambda = -\frac{1}{\pi}$ and $f(x) = 1$. 

\[ y(x) = 1 + \sin x - \cos x \text{ for } \lambda = -\frac{1}{2} \text{ and } f(x) = 1. \]

No, the answer is incorrect.

Score: 0

Accepted Answers:
\[ y(x) = x + \frac{4}{3} (2 \cos x + \sin x) \text{ for } \lambda = \frac{3}{2} \text{ and } f(x) = x. \]

7) Consider the differential equation \(2y''(x) + y'(x) + xy(x) = 0\), and let the function \(v(x)\) be defined as
\[ v(x)[2y''(x) + y'(x) + xy(x)] = [A(x)y'(x) + B(x)y(x)]', \]
for some twice differentiable functions \(A(x)\) and \(B(x)\). Then

- \( v(x) \) is a solution of the equation \(2y''(x) + y'(x) + xy(x) = 0 \)

- \( v(x) \) is a solution of the equation \(2y''(x) - y'(x) + xy(x) = 0 \)

- \( v(x) \) is a solution of the equation \(2y''(x) + xy(x) = 0 \)

- none of the above.

No, the answer is incorrect.

Score: 0

Accepted Answers:
\[ v(x) \text{ is a solution of the equation } 2y''(x) - y'(x) + xy(x) = 0 \]

8) Consider the Fredholm integral equation \( y(x) = x + \lambda \int_0^1 e^{x+t} y(t) \, dt. \) Then it has a solution

- \( y(x) = x + \frac{2e^x}{3-e^x} \text{ for } \lambda = \frac{1}{e^x-1} \)

- \( y(x) = x + \frac{2e^x}{3-e^x} \text{ for } \lambda = 1 \)

- \( y(x) = x + \frac{2e^x}{3-e^x} \text{ for } \lambda = 1 \)

- \( y(x) = x + \frac{2e^x}{3-e^x} \text{ for } \lambda = -1. \)

No, the answer is incorrect.

Score: 0

Accepted Answers:
\[ y(x) = x + \frac{2e^x}{3-e^x} \text{ for } \lambda = 1 \]

9) Consider the differential equation \( x^2 y'' + xy' - 4y = 0 \) with appropriate boundary conditions. Then the adjoint equation is given by

- \( x^2 v'' + 3x v' - 3v = 0 \)

- \( x^2 v'' - 3x v' - 4v = 0 \)

- \( x^2 v'' - 2v' - 4v = 0 \)

- None of these.

No, the answer is incorrect.

Score: 0

Accepted Answers:
\[ x^2 v'' + 3x v' - 3v = 0 \]
10) Consider the boundary value problem \(y''(x) + 2y'(x) + y(x) = f(x)\) with the boundary conditions \(y(0) = 0 = y(1)\). Then the Green's function \(G(\xi, x)\) for the given boundary value problem is

- \[G(\xi, x) = \begin{cases} \xi(x-1)e^{\xi-x}, & 0 \leq \xi \leq x; \\ xe^{\xi-x}(\xi-1), & x < \xi \leq 1. \end{cases}\]
- \[G(\xi, x) = \begin{cases} (\xi-1)xe^{\xi-x}, & 0 \leq \xi \leq x; \\ xe^{\xi-x}(x-1), & x < \xi \leq 1. \end{cases}\]
- \[G(\xi, x) = \begin{cases} (\xi-1)e^{\xi-x} \xi, & 0 \leq \xi \leq x; \\ e^{\xi-x}e^{\xi-1}x, & x < \xi \leq 1. \end{cases}\]

No, the answer is incorrect.
Score: 0
Accepted Answers:
None of these.