Week 3: Assignment 3

The due date for submitting this assignment has passed.
As per our records, you have not submitted this assignment.

1. Which of the following sets are always connected in any metric space?
   - Any connected component.
   - Singleton sets.
   - Any subset that is closed and open.
   - Points sets.
   - No, the answer is incorrect.
   - Score: 0
   Accepted Answers:
   - Any connected component.
   - Singleton sets.

2. Let \( X \) be a compact metric space. Which of the following spaces of continuous functions to the real numbers on \( X \) is always equicontinuous?
   - Any finite collection of continuous functions.
   - Any countable collection of continuous functions.
   - The space of all bounded and continuous functions.
   - No, the answer is incorrect.
   - Score: 0
   Accepted Answers:
   - Any finite collection of continuous functions.
   - Any countable collection of continuous functions.

3. Consider the function
   \[
   f(x) = \begin{cases} 
   \sin(1/x) & \text{if } x \neq 0, \\
   y & \text{if } x = 0. 
   \end{cases}
   \]
   Here \( y \) is a real number.
   - For which choice(s) of \( y \) is the graph of the function a connected subset of \( \mathbb{R}^2 \)?
     - \( y = 0 \)
     - \( y = 1 \)
     - \( y = 0 \)
     - For no choice of \( y \) is the graph of \( f(x) \) connected.
     - No, the answer is incorrect.
     - Score: 0
     Accepted Answers:
     - \( y = 0 \)
     - \( y = 1 \)
     - \( y = 0 \)

4. Which of the following subspaces of the space \( C([0, 1], \mathbb{R}) \) are dense?
   - The collection of differentiable functions.
   - The collection of all polynomials with integer coefficients.
   - The collection of all polynomials with rational coefficients.
   - No, the answer is incorrect.
   - Score: 0
   Accepted Answers:
   - The collection of differentiable functions.
   - The collection of all polynomials with rational coefficients.

5. Consider the sequence \( x_n \). Which of the following statements are true?
   - The sequence converges.
   - \( \lim_{n \to \infty} x_n = 1 \)
   - \( \lim_{n \to \infty} x_n = 0 \)
   - \( \lim_{n \to \infty} x_n = -1 \)
   - No, the answer is incorrect.
   - Score: 0
   Accepted Answers:
   - \( \lim_{n \to \infty} x_n = 1 \)
   - \( \lim_{n \to \infty} x_n = 0 \)
   - \( \lim_{n \to \infty} x_n = -1 \)

6. Which of the following statements about connectedness and path-connectedness in a metric space are true?
   - Any path-connected set is always connected.
   - Any path-connected component is always closed.
   - Any connected component is always closed.
   - No, the answer is incorrect.
   - Score: 0
   Accepted Answers:
   - Any path-connected set is always connected.
   - Any connected component is always closed.
   - Any path-connected component is always closed.

7. Let \( X \) and \( Y \) be two continua. Which of the following statements are true?
   - If \( f \) is a homeomorphism of \( X \), then \( f^{-1} \) is also a homeomorphism.
   - If \( X \) and \( Y \) are homeomorphic, then we can find a homeomorphism from \( X \) onto \( Y \).
   - Hint: Use connectedness and compactness.
   - No, the answer is incorrect.
   - Score: 0
   Accepted Answers:
   - If \( X \) and \( Y \) are homeomorphic, then we can find a homeomorphism from \( X \) onto \( Y \).
   - If \( f \) is a homeomorphism of \( X \), then \( f^{-1} \) is also a homeomorphism.